HOMEWORK ASSIGNMENT 9 (Optional, non-graded)

All the problems in this homework are from W. Strauss book, except the last one.

1. Section 6.1 (page 160): # 1 (Optional)
2. Section 6.1 (page 160): # 4 (This could be in Chapter 1)
3. Section 6.1 (page 160): # 6 (This could be in Chapter 1)
4. Section 6.2 (page 165): # 2
5. Section 6.2 (page 165): # 4
6. Section 6.2 (page 165): # 6
7. Section 6.2 (page 165): # 7
8. Section 6.3 (page 172): # 1
9. Section 6.3 (page 172): # 2
10. (Last year Practice Exam for Final exam)

    Part a. Let $u \geq 0$ and $\Delta u = 0$ in the unit disk $D = \{(x, y) : x^2 + y^2 \leq 1\}$. Using the Mean-Value Property for harmonic functions, prove the following version of the so-called Harnack inequality

    $$\frac{1-r}{1+r} u(0,0) \leq u(x,y) \leq \frac{1+r}{1-r} u(0,0),$$

    where $r = \sqrt{x^2 + y^2} < 1$.

    Part b. Consider the following problem

    $$\begin{aligned} &\Delta u = 0, \quad D = \{(x, y) : x^2 + y^2 \leq 1\}, \\ &u = h, \quad \text{on } \partial D. \end{aligned}$$

    Part b.1. Show that if $h \geq 0$ then $u > 0$ unless $h \equiv 0$.

    Part b.2. Let $u(0) = 1$ and $h \geq 0$. Show that

    $$\frac{1}{3} \leq u(x,y) \leq 3 \quad \text{for all } x^2 + y^2 = \frac{1}{4}.$$