

HOMWORK ASSIGNMENT 5, Math 241, Section 002

Name:

Due: Friday February 28, 9pm

1. Use the method of separation of variables to find the solution $u(x, t)$ to the following modified wave equation

$$u_{tt} = 2u_{xx} - u, \quad 0 < x < 1, \quad t > 0,$$

with boundary conditions

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0,$$

and initial conditions

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x).$$

2. Determine all scalars λ and non-zero solutions $\phi(x)$, given the ODE

$$\phi''(x) + \lambda^2\phi(x) = 0,$$

for $0 < x < 1$, and also given two boundary conditions: $\phi(0) + \phi(1) = 0$, and $\phi'(1) = 0$.

Remark: While the BCs are *non-standard* (none of the three types), the procedure to solve the problem is the usual one.

3. (**Optional** if your grade on P3 of Midterm 1 is above 12, mandatory otherwise)

Solve the Laplace equation on a rectangle

$$u_{xx} + u_{yy} = 0, \quad 0 < x < a, \quad 0 < y < b,$$

with the following boundary conditions:

$$u(0, y) = 1, \quad u(a, y) = 0, \quad 0 < y < b,$$

and

$$u_y(x, 0) = 0, \quad u_y(x, b) = 1, \quad 0 < x < a.$$

4. Read Sections 4.4 (also 4.2, 4.5) and 5.3 of R. Haberman's book.