HOMEWORK ASSIGNMENT 4, Math 425

Name: **Due:** Friday February 14, 8PM

Problem 0: Strauss, Section 2.4 #16, p.54

Solve the diffusion equation with constant dissipation:

$$u_t - u_{xx} + bu = 0 \text{ for } -\infty < x < \infty, \text{ with } u(x,0) = \phi(x),$$

where $b > 0$ is a constant. (Hint: Make the change of variables $u(x,t) = e^{-bt}v(x,t)$.)

Problem 1: Strauss, Section 2.4 #1, p.52

Solve the diffusion equation with the initial condition

$$\phi(x) = 1 \quad \text{for } |x| < l \qquad \text{and} \qquad \phi(x) = 0 \quad \text{for } |x| > l.$$

Write your answer in terms of $\text{Erf}(x)$.

Problem 2: Strauss, Section 2.4 #6, p.52

Compute $\int_0^\infty e^{-x^2} dx$. Hint: This is a function that cannot be integrated by formula. So use the following trick: Transform the double integral $\int_0^\infty e^{-x^2} dx \int_0^\infty e^{-y^2} dy$ into polar coordinates and you'll end up with a function that can be integrated easily.

Problem 3: Strauss, Section 2.4 #11, p.53

a) Consider the heat equation on the whole line with the usual initial condition $u(x,0) = \phi(x)$. Show that, if $\phi(x)$ is and odd function, then the solution $u(x,t)$ is also an odd function of x. Hint: Consider $u(-x,t) + u(x,t)$ and use uniqueness.

b) Show that the same is true if odd is replaced by even.

c) Show that the analogous statements are true for the wave equation.

Problem 4 Strauss, Section 2.4 #17, p.54

Solve the diffusion equation with variable dissipation:

$$u_t - ku_{xx} + bt^2 u = 0 \quad \text{for } -\infty < x < \infty \quad \text{with } u(x,0) = \phi(x),$$

where $b > 0$ is a constant. Hint: The solutions of the ODE $w_t + bt^2 w = 0$ are $Ce^{-bt^3/3}$. So make the change of variables $u(x,t) = e^{-bt^3/3}v(x,t)$ and derive an equation for v.

Problem 5: Strauss, Section 2.4 #19, p.54

(a) Show that $S_2(x, t, t) = S(x, t)S(y, t)$ satisfies the two-dimensional diffusion equation: $S_t = k(S_{xx} + S_{yy})$.

(b) Deduce that $S_2(x, y, t)$ is the source function for two-dimensional diffusions (i.e., that the general solution is given as convolutions with this source).

Problem 6: Strauss, Section 3.1 #1, p.60

Solve $u_t = ku_{xx}$, $u(x, 0) = e^{-x}$, $u(0, t) = 0$ on the half-line $0 < x < \infty$.

Problem 7: Strauss, Section 3.3 #1, p.70

Solve the inhomogeneous diffusion problem on the half-line

$$\begin{aligned} &v_t - kv_{xx} = f(x, t) \qquad &&\text{for } 0 < x < \infty,\ 0 < t < \infty, \\ &v(0, t) = 0, &&v(x, 0) = \phi(x). \end{aligned}$$

Problem 8:

Review all the material so far for the first midterm (Lecture notes 1 to 8).