HOMEWORK ASSIGNMENT 3

Name: **Due:** Saturday February 8, 8PM

All the problems in this homework are from W. Strauss book. The level of difficulty is marked as *,**,***.

Problem 1*: Strauss, Section 2.1 #2, p.38

Solve $u_{tt} = c^2 u_{xx}$, $u(x, 0) = \log(1 + x^2)$, $u_t(x, 0) = 4 + x$.

Problem 2*: Strauss, Section 2.1 #7, p.38

If both ϕ and ψ are odd functions of x, show that the solution $u(x, t)$ of the wave equation is also odd in x for all t.

Problem 3**: Strauss, Section 2.1 #10, p.38

Solve $u_{xx} + u_{xt} - 20u_{tt} = 0$, $u(x, 0) = \phi(x)$, $u_t(x, 0) = \psi(x)$.

Problem 4**: Strauss, Section 2.1 #11, p.38

Find the general solution of $3u_{tt} + 10u_{xt} + 3u_{xx} = \sin(x + t)$.

Problem 5*: Strauss, Section 2.2 #1, p.41

Use the energy conservation of the wave equation to prove that the only solution with $\phi \equiv 0$ and $\psi \equiv 0$ is $u \equiv 0$. (Hint: Use the first vanishing theorem in Section A.1.)

Problem 6**: Strauss, Section 2.2 #5, p.41

For the *damped* string (equation below), show that the energy (defined as for the usual wave equation) decreases:

$$u_{tt} - c^2 u_{xx} + r u_t = 0, \qquad r > 0.$$

(Hint: Two possible ways: 1) Proceed as in class, 2) Multiply the equation by u_t, identify the derivative of a square and integrate by parts).

Problem 7**: Strauss, Section 2.3 #4, p.46

Consider the diffusion equation $u_t = u_{xx}$ in $\{0 < x < 1, 0 < t < \infty\}$ with $u(0,t) = u(1,t) = 0$ and $u(x,0) = 4x(1-x)$.

a) Show that $0 < u(x,t) < 1$ for all $t > 0$ and $0 < x < 1$.

b) Show that $u(x,t) = u(1-x,t)$ for all $t \geq 0$ and $0 \leq x \leq 1$.

c) Use the energy method to show that $\int_0^1 u^2(x,t)dx$ is a strictly decreasing function of t.

Problem 8**: Strauss, Section 2.3 #6, p.46

Prove the *comparison principle* for the diffusion equation: If u and v are two solutions, and if $u \leq v$ for $t = 0$, for $x = 0$ and for $x = l$, then $u \leq v$ for $0 \leq x \leq l$, $0 \leq t < \infty$.

Problem 9**: Strauss, Section 2.4 #15, p.53

Prove the uniqueness of the diffusion problem with Neumann boundary conditions by the energy method:

$$u_t - u_{xx} = f(x,t) \text{ for } 0 < x < l, t > 0, u(x,0) = \phi(x), u_x(0,t) = g(t), u_x(l,t) = h(t).$$

Problem 10**: Strauss, Section 2.4 #16, p.54

Solve the diffusion equation with constant dissipation:

$$u_t - u_{xx} + bu = 0 \text{ for } -\infty < x < \infty, \text{ with } u(x,0) = \phi(x),$$

where $b > 0$ is a constant. (Hint: Make the change of variables $u(x,t) = e^{-bt}v(x,t)$.)

Problem 11:

Read sections 2.3, 2.4, 2.5, and 3.1 of W. Strauss book.