HOMEWORK ASSIGNMENT 1

Name: **Due:** Thursday January 23, 1PM

These problems correspond to Review of ODEs and Sections 1.1, 1.2 of the book.
The level of difficulty is marked as *,**,***.

Problem 1*:

Determine if the following differential equations are linear and find their general solution:

a) $2y''(t) + 5y'(t) + 2y(t) = 0$,

b) $x'(t) + x^2(t)\sin(t) = 0$.

Problem 2*:

Solve the following initial value problems:

a) $x'(t) + x(t)\cos(t) = 0$, with $x(\pi) = 10$,

b) $5y''(t) + 8y'(t) + 5y(t) = 0$, with $y(0) = 1$, $y'(0) = 0$,

c) $x'(t) = x(t) - 4y(t)$, $y'(t) = x(t) + y(t)$, with $x(0) = 1$, $y(0) = 1$.

Problem 3**:

Prove that for $c < 0$ the solution of the initial value problem $y''(t) + cy(t) = 0$ with $y(0) = a$, $y'(0) = b$, exists and is unique. (Hint: Factor the operator and then apply twice the uniqueness theorem from the notes).

Problem 4***:

Torricelli's law states that fluid will leak out of a small hole at the base of a container at a rate proportional to the square root of the height of the fluid's surface from the base.

a) Suppose that a cylindrical container is initially filled to a depth of one foot. If it takes one minute for three quarters of the fluid to leak out, how long will it take for all of the fluid to leak out?

b) It is desired to design a *water clock* by making a container that is in the shape of some surface of revolution with a small hole in the bottom, so that as the water empties out of the hole, the water level in the container falls at a constant rate. What should be the shape of the container?

Problem 5***:

For a function $u(x, y)$ of two variables, its Laplacian is defined to be $\Delta u = u_{xx} + u_{yy}$. Which radial functions (i.e., functions of the polar coordinate r but independent of θ) are harmonic (i.e., satisfy the PDE $\Delta u = 0$)?

Problem 6*:

Show that the difference of two solutions of an inhomogeneous linear equation $\mathcal{L}u = g$ with the same g is a solution of the homogeneous equation $\mathcal{L}u = 0$.

Problem 7* :

Verify by direct substitution that $u_n(x, y) = \sin(nx) \sinh(ny)$ is a solution of $u_{xx}+u_{yy} = 0$ for every $n > 0$.

Problem 8*:

Solve the first-order equation $2u_t + 3u_x = 0$ with the condition $u = \sin(x)$ when $t = 0$.

Problem 9*:

Solve the equation $u_x + u_y = 1$ using the coordinate method.

Problem 10:

Read the Review of ODEs notes and sections 1.1 to 1.5 of W. Strauss book. Which part was most confusing to you?