HOMEWORK ASSIGNMENT 1

Name: **Due:** Thursday January 23, 10am

Problem 1:

Find the general solutions to the ODEs:

1. $y' + 5y = 0$,
2. $xy' + 3x^3 y = 0$,
3. $y'' + 4y = 0$,
4. $y'' - 4y = 0$,
5. $y'' - 4y' + 4y = 0$.

Problem 2:

Let V be a vector space equipped with an inner product $\langle v_i, v_j \rangle$ and a basis $\mathcal{B} = \{v_1, \ldots, v_n\}$. The basis vectors are orthogonal if $\langle v_i, v_j \rangle = 0$ for any $i \neq j$. Let $w \in V$ have the expansion $w = c_1 v_1 + \cdots + c_n v_n$. In general, solving for c_i requires row reduction.

a) If the basis vectors are orthogonal, there is an explicit formula for the c_i. Show that

$$c_i = \frac{\langle w, v_i \rangle}{\langle v_i, v_i \rangle}, \qquad i = 1, \ldots, n.$$

b) In the case where $V = \mathbb{R}^3$, $\mathcal{B} = \{v_1, v_2, v_3\} = \{(1,0,0), (0,1,1), (0,1,-1)\}$, use this to find the coefficients c_1, c_2, c_3, where $w = (3,4,5)$.

Problem 3:

The following table gives the inner product of vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

	u	**v**	**w**
u	9	0	6
v	0	1	3
w	6	3	38

For example, $\mathbf{v} \cdot \mathbf{w} = 3$.

a) Find a unit vector in the same direction as $\mathbf{u}$.

b) Compute $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$.

c) Compute the length $\|\mathbf{v}+\mathbf{w}\|$.

d) Find the orthogonal projection of $\mathbf{w}$ into the plane E spanned by $\mathbf{u}$ and $\mathbf{v}$. Express your solution as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

e) Find a unit vector orthogonal to the plane E. Express your solution as a linear combination of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

f) Find an orthonormal basis of the three dimensional space spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$. Here, orthonormal basis means the basis vectors are orthogonal units vectors. Express the solution as linear combinations of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$. (Gram-Schmidt algorithm)

Problem 4:

Show that if $u(x,t)$ is a solution to the transport equation $u_t = u_x$, then it is also a solution of the wave equation $u_{tt} = u_{xx}$. Verify that $u(x,t) = f(x+t)$ is always a solution to the transport equation $u_t = u_x$.

Problem 5:

Let f, g be functions of three variables, and $\vec{F}$ a vector field in $\mathbb{R}^3$.

a) Prove the following product rule by direct computation:

$$\mathrm{div}(f\vec{F}) = \nabla f \cdot \vec{F} + f\mathrm{div}\vec{F}.$$

b) Use the product rule and the divergence theorem to show the following integration by parts formula:

$$\int_\Omega f\Delta g = \int_{\partial\Omega} f\frac{\partial g}{\partial n} - \int_\Omega \nabla f \cdot \nabla g,$$

where Ω is a bounded region in $\mathbb{R}^3$, $\vec{n}$ is the outward unit normal vector to the boundary $\partial\Omega$, and $\frac{\partial g}{\partial n} = \vec{n} \cdot \nabla g$ is the directional derivative in the normal direction.

Problem 6:

Exercise 1.2.1 of the book (*Applied PDE, with Fourier Series and Boundary Value Problems*, Fifth Edition. Richard Haberman).

Problem 7:

Exercise 1.2.2 of the book (*Applied PDE, with Fourier Series and Boundary Value Problems*, Fifth Edition. Richard Haberman).

Problem 8:

Exercise 1.2.3 of the book (*Applied PDE, with Fourier Series and Boundary Value Problems*, Fifth Edition. Richard Haberman).

Problem 9:

Exercise 1.2.4 of the book (*Applied PDE, with Fourier Series and Boundary Value Problems*, Fifth Edition. Richard Haberman).

Problem 10:

Read Chapter 1 and Section 2.2 of Richard Haberman's book.