HOMEWORK ASSIGNMENT 10

Name: **Due:** Friday December 6, 6pm.

Note: Homework must be submitted online on Canvas (scanned).

PROBLEM 1

Find the coefficients for the model below that best fit the data $t = 0, 1, 4$, $y = 0, 1, 0$ in the least squares sense:

$$y = a + b\sqrt{t}.$$

PROBLEM 2:

Given the input data $x = 0, 1, 1, 2$, $y = 0, 1, 1, 3$ and the output data $z = 0, 1, 2, 2$,

1. Find the plane that best fits the data in the least squares sense.
2. Find the paraboloid $z = ax^2 + by^2 + c$ that best fits the data in the least squares sense.

PROBLEM 3

1. Given the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ -2 & 1 & 2 \\ 3 & -1 & 3 \end{bmatrix}$, find $A\vec{u}$ for a) $\vec{u} = [1, -3, 2]^T$, b) $\vec{u} = [3, 0, -2]^T$.
2. Given $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 1 & 5 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 1 \\ 6 & -3 \\ 1 & -2 \end{bmatrix}$, find the matrices AB and BA.
3. Given $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, find A^2, A^3, and then find a formula for A^n.

PROBLEM 4

Find a scalar multiple of each vector which is a probability vector:

1. $\vec{u} = [3, 0, 2, 5, 3]^T$,
2. $\vec{v} = [2, 1/2, 0, 1/4, 1]^T$.

Problem 5

Find the unique fixed probability vector of each matrix:

1. $A = \begin{bmatrix} 0.2 & 0.5 \\ 0.8 & 0.5 \end{bmatrix}$,

2. $B = \begin{bmatrix} 0 & 1/2 & 0 \\ 3/4 & 1/2 & 1 \\ 1/4 & 0 & 0 \end{bmatrix}$.

Problem 6

For a Markov chain, the transition matrix is $P = \begin{bmatrix} 1/2 & 3/4 \\ 1/2 & 1/4 \end{bmatrix}$ with initial state $\vec{u}_0 = [3/4, 1/4]^T$. Find $\vec{u}(2)$, the steady state vector and the matrix P^n approaches as $n \to \infty$.

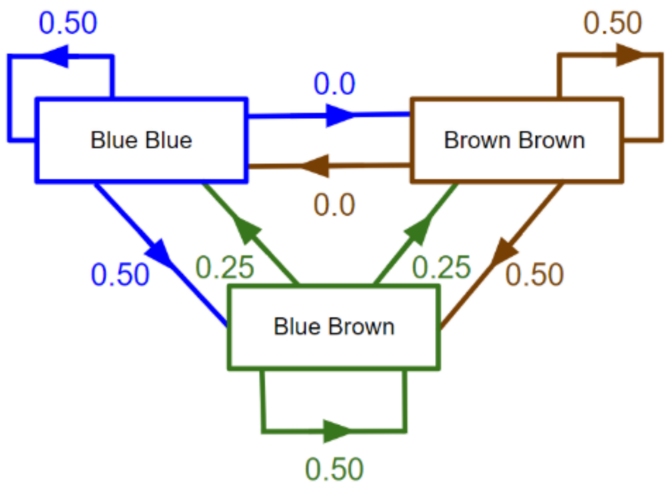
Problem 7

By analyzing the behavior of the value of a stock of company A during a year, we found that: if its value went up on a given day, then 70% of the times it went up again the next day. However, if it went down, then 40% of the times it went up the next day. We are asked to construct a Markov chain model to predict the value of that stock.

1. Write the stochastic matrix A that defines the Markov chain.
2. If today the value of the stock went up, what is the probability that it will go down *the day after tomorrow*?
3. If the value of the stock went down today, which fraction of days will the value go up *in the long term*? What if today it went up?

Problem 8

Simple Mendelian genetics assumes that a trait, such as eye color, is determined by a combination of two genes, one inherited from each parent. The probability of a parent with a specific pair of genes having offspring with each gene combination can be illustrated by a transition diagram. Below is the transition diagram for eye color with Blue and Brown as the two possible genes.



1. Find the stochastic matrix M that models this Markov chain.

2. What is probability that the grandchildren of a parent with the gene pair Brown Brown has the gene Blue Blue?

3. In a population which currently only has Blue and Brown genes for eye color, which combination will be most common after a long period of time? Compute the fraction of people with that gene (the most common in the long term).