Supplementary Problems

FACTORIAL NOTATION AND BINOMIAL COEFFICIENTS

2.38. Find: (*a*) 10!, 11!, 12! (*b*) 60! (*Hint*: Use Stirling's approximation to $n!$.)

2.39. Compute: (*a*) $\dfrac{16!}{14!}$, (*b*) $\dfrac{14!}{11!}$, (*c*) $\dfrac{8!}{10!}$, (*d*) $\dfrac{10!}{13!}$.

2.40. Simplify: (*a*) $\dfrac{(n+1)!}{n!}$, (*b*) $\dfrac{n!}{(n-2)!}$, (*c*) $\dfrac{(n-1)!}{(n+2)!}$, (*d*) $\dfrac{(n-r+1)!}{(n-r-1)!}$.

2.41. Compute: (*a*) $\dbinom{5}{2}$, (*b*) $\dbinom{7}{3}$, (*c*) $\dbinom{14}{2}$, (*d*) $\dbinom{6}{4}$, (*e*) $\dbinom{20}{17}$, (*f*) $\dbinom{18}{15}$.

2.42. Show that: (*a*) $\dbinom{n}{0}+\dbinom{n}{1}+\dbinom{n}{2}+\dbinom{n}{3}+\cdots+\dbinom{n}{n}=2^n$,

(*b*) $\dbinom{n}{0}-\dbinom{n}{1}+\dbinom{n}{2}-\dbinom{n}{3}+\cdots\pm\dbinom{n}{n}=0$.

2.43. Evaluate the following multinomial coefficients (defined in Problem 2.36):

(*a*) $\dbinom{6}{2,3,1}$, (*b*) $\dbinom{7}{3,2,2,0}$, (*c*) $\dbinom{9}{3,5,1}$, (*d*) $\dbinom{8}{4,3,2}$.

2.44. Find the (*a*) ninth and (*b*) tenth rows of Pascal's triangle, assuming the following is the eighth row:

1 8 28 70 56 28 8 1

COUNTING PRINCIPLES, SUM AND PRODUCT RULES

2.45. A store sells clothes for men. It has 3 different kinds of jackets, 7 different kinds of shirts, and 5 different kinds of pants. Find the number of ways a person can buy:

(*a*) one of the items for a present, (*b*) one of each of the items for a present.

2.46. A restaurant has, on its dessert menu, 4 kinds of cakes, 2 kinds of cookies, and 3 kinds of ice cream. Find the number of ways a person can select: (*a*) one of the desserts, (*b*) one of each kind of dessert.

2.47. A class contains 8 male students and 6 female students. Find the number of ways that the class can elect: (*a*) a class representative; (*b*) 2 class representatives, 1 male and 1 female; (*c*) a president and a vice-president.

2.48. Suppose a password consists of 4 characters where the first character must be a letter of the (English) alphabet, but each of the other characters may be a letter or a digit. Find the number of:

(*a*) passwords, (*b*) passwords beginning with one of the 5 vowels.

2.49. Suppose a code consists of 2 letters followed by 3 digits. Find the number of:

(*a*) codes, (*b*) codes with distinct letters, (*c*) codes with the same letters.

2.50. There are 6 roads between A and B and 4 roads between B and C. Find the number n of ways a person can drive: (*a*) from A to C by way of B, (*b*) round-trip from A to C by way of B, (*c*) round-trip from A to C by way of B without using the same road more than once.

PERMUTATIONS AND ORDERED SAMPLES

2.51. Find the number n of ways a judge can award first, second, and third places in a contest with 18 contestants.

2.52. Find the number n of ways 6 people can ride a toboggan where: (*a*) anyone can drive, (*b*) one of 3 must drive.

2.53. A debating team consists of 3 boys and 3 girls. Find the number n of ways they can sit in a row where: (*a*) there are no restrictions, (*b*) the boys and girls are each to sit together, (*c*) just the girls are to sit together.

2.54. Find the number n of permutations that can be formed from all the letters of each word: (*a*) QUEUE, (*b*) COMMITTEE, (*c*) PROPOSITION, (*d*) BASEBALL.

2.55. Find the number n of different signals, each consisting of 8 flags hung in a vertical line, that can be formed from 4 identical red flags, 2 identical blue flags, and 2 identical green flags.

2.56. Find the number n of ways 5 large books, 4 medium-size books, and 3 small books can be placed on a shelf so that all books of the same size are together.

2.57. A box contains 12 light bulbs. Find the number n of ordered samples of size 3:

(*a*) with replacement, (*b*) without replacement.

2.58. A class contains 10 students. Find the number n of ordered samples of size 4:

(*a*) with replacement, (*b*) without replacement.

COMBINATIONS

2.59. A restaurant has 6 different desserts. Find the number of ways a customer can choose 2 of the desserts.

2.60. A store has 8 different mystery books. Find the number of ways a customer can buy 3 of the books.

2.61. A box contains 6 blue socks and 4 white socks. Find the number of ways two socks can be drawn from the box where: (*a*) there are no restrictions, (*b*) they are different colors, (*c*) they are to be the same color.

2.62. A class contains 9 boys and 3 girls. Find the number of ways a teacher can select a committee of 4.

2.63. Repeat Problem 2.62, but where: (*a*) there are to be 2 boys and 2 girls, (*b*) there is to be exactly 1 girl, (*c*) there is to be at least 1 girl.

2.64. A woman has 11 close friends. Find the number of ways she can invite 5 of them to dinner.

2.65. Repeat Problem 2.64, but where 2 of the friends are married and will not attend separately.

2.66. Repeat Problem 2.64, but where 2 of the friends are not on speaking terms and will not attend together.

2.67. A person is dealt a poker hand (5 cards) from an ordinary deck with 52 cards. Find the number of ways the person can be dealt: (*a*) four of a kind, (*b*) a flush.

2.68. A student must answer 10 out of 13 questions. (*a*) How many choices are there? (*b*) How many if the student must answer the first 2 questions? (*c*) How many if the student must answer the first or second question but not both?

PARTITIONS

2.69. Find the number of ways 6 toys may be divided evenly among 3 children.

2.70. Find the number of ways 6 students can be partitioned into 3 teams containing 2 students each. (Compare with Problem 2.69.)

2.71. Find the number of ways 6 students can be partitioned into 2 teams where each team contains 2 or more students.

2.72. Find the number of ways 9 toys may be divided among 4 children if the youngest is to receive 3 toys and each of the others 2 toys.

2.73. There are 9 students in a class. Find the number of ways the students can take 3 tests if 3 students are to take each test.

2.74. There are 9 students in a class. Find the number of ways the students can be partitioned into 3 teams containing 3 students each. (Compare with Problem 2.73.)

TREE DIAGRAMS

2.75. Teams A and B play in the world series of baseball where the team that first wins 4 games wins the series. Suppose A wins the first game and that the team that wins the second game also wins the fourth game. (*a*) Find the number n of ways the series can occur, and list the n ways the series can occur. (*b*) How many ways will B win the series? (*c*) How many ways will the series last 7 games?

2.76. Suppose $A, B, \ldots, F$ in Fig. 2-7 denote islands, and the lines connecting them bridges. A person begins at A and walks from island to island. The person stops for lunch when he or she cannot continue to walk without crossing the same bridge twice. (*a*) Construct the appropriate tree diagram, and find the number of ways the person can walk before eating lunch. (*b*) At which islands can he or she eat lunch?

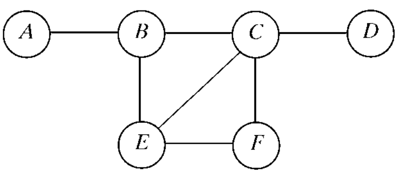

Fig. 2-7

Answers to Supplementary Problems

2.38. (*a*) 3,628,800; 39,916,800; 479,001,600. (*b*) log(60!) = 81.92, so $60! \approx 6.59 \times 10^{81}$.

2.39. (*a*) 240; (*b*) 2184; (*c*) 1/90; (*d*) 1/1716.

2.40. (*a*) $n + 1$; (*b*) $n(n-1) = n^2 - n$; (*c*) $1/[n(n+1)(n+2)]$; (*d*) $(n-r)(n-r+1)$.

2.41. (*a*) 10; (*b*) 35; (*c*) 91; (*d*) 15; (*e*) 1140; (*f*) 816.

2.42. *Hint*: Expand (*a*) $(1+1)^n$; (*b*) $(1-1)^n$.

2.43. (*a*) 60; (*b*) 210; (*c*) 504; (*d*) Not defined.

2.44. (*a*) 1, 9, 36, 84, 126, 126, 84, 36, 9, 1; (*b*) 1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1.

2.45. (*a*) 15; (*b*) 105.

2.46. (*a*) 9; (*b*) 24.

2.47. (*a*) 14; (*b*) 48; (*c*) 182.

2.48. (*a*) $26 \cdot 36^3$; (*b*) $5 \cdot 36^3$.

2.49. (*a*) $26^2 \cdot 10^3 = 676{,}000$; (*b*) $26 \cdot 25 \cdot 10^3 = 650{,}000$; (*c*) $26 \cdot 10^3 = 26{,}000$.

2.50. (*a*) 24; (*b*) $24^2 = 576$; (*c*) 360.

2.51. $n = 18 \cdot 17 \cdot 16 = 4896$.

2.52. (*a*) $6! = 720$; (*b*) $3 \cdot 5! = 360$.

2.53. (*a*) $6! = 720$; (*b*) $2 \cdot 3! \cdot 3! = 72$; (*c*) $4 \cdot 3! \cdot 3! = 144$.

2.54. (*a*) 30; (*b*) $\dfrac{9!}{2!2!2!} = 45{,}360$; (*c*) $\dfrac{11!}{2!3!2!} = 1{,}663{,}200$; (*d*) $\dfrac{8!}{2!2!2!} = 5040$.

2.55. $n = \dfrac{8!}{4!2!2!} = 420$.

2.56. $3!5!4!3! = 103{,}680$.

2.57. (*a*) $12^3 = 1728$; (*b*) 1320.

2.58. (*a*) $10^4 = 10{,}000$; (*b*) $10 \cdot 9 \cdot 8 \cdot 7 = 5040$.

2.59. $C(6,2) = 15$.

2.60. $C(8,3) = 56$.

2.61. (*a*) $C(10,2) = 45$; (*b*) $6 \cdot 4 = 24$; (*c*) $C(6,2) + C(4,2) = 21$ or $45 - 24 = 21$.

2.62. $C(12,4) = 495$.

2.63. (*a*) $C(9,2) \cdot C(3,2) = 108$; (*b*) $C(9,3) \cdot 3 = 252$;
(*c*) $9 + 108 + 252 = 369$ or $C(12,4) - C(9,4) = 495 - 126 = 369$.

2.64. $C(11,5) = 462$.

2.65. 210.

2.66. 252.

2.67. (*a*) $13 \cdot 48 = 624$; (*b*) $4 \cdot C(13, 5) = 5148$.

2.68. (*a*) $C(13, 10) = C(13, 3) = 286$; (*b*) $2 \cdot C(11, 9) = 2 \cdot C(11, 2) = 110$.

2.69. 90.

2.70. 15.

2.71. (*Hint*: The number of subsets excluding $\varnothing$ and the 6 singleton subsets.) $2^5 - 1 - 6 = 25$.

2.72. $\dfrac{9!}{3!2!2!2!} = 7560$.

2.73. $\dfrac{9!}{3!3!3!} = 1680$.

2.74. $\dfrac{1680}{3!} = 280$.

2.75. Construct the appropriate tree diagram as in Fig. 2-8. Note that the tree begins at A, the winner of the first game, and that there is only one choice in the fourth game, the winner of the second game. (*a*) The diagram shows that $n = 15$ and that the series can occur in the following 15 ways:

AAAA, *AABAA*, *AABABA*, *AABABBA*, *AABABBB*, *ABABAA*, *ABABABA*, *ABABABB*, *ABABBAA*, *ABABBAB*, *ABABBB*, *ABBBAAA*, *ABBBAAB*, *ABBBAB*, *ABBBB*

(*b*) 6; (*c*) 8.

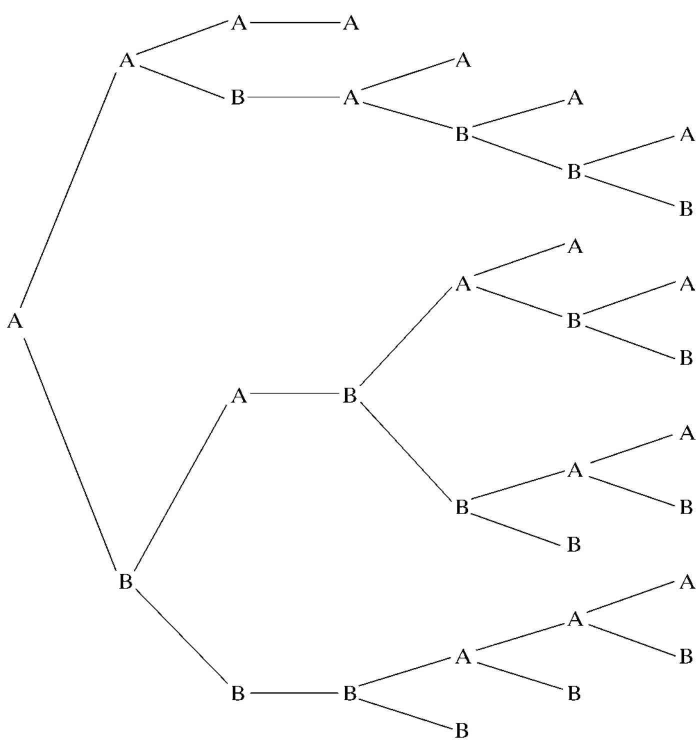

Fig. 2-8

2.76. (*a*) See Fig. 2-9. There are 11 ways to take his walk. (*b*) B, D, or E.

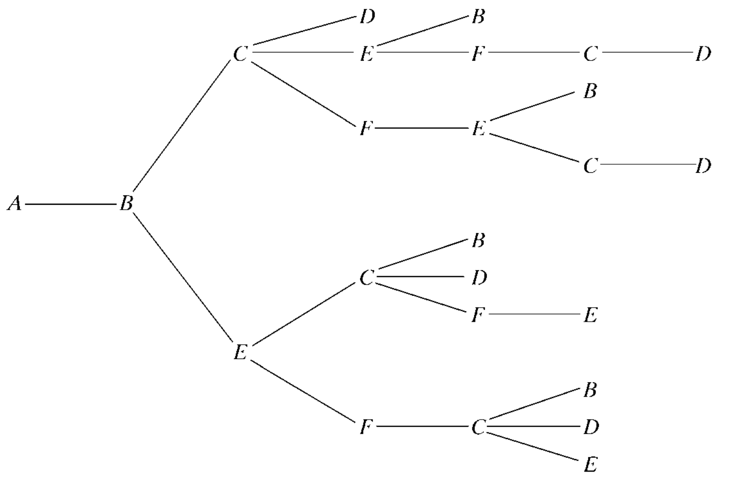

Fig. 2-9