Finding Maxima and Minima

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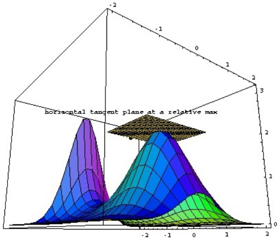
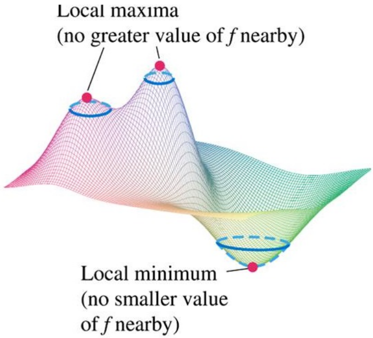
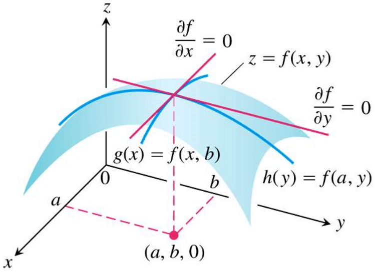




The tangent plane of such a point will be horizontal.

What does that say about the partial derivatives?



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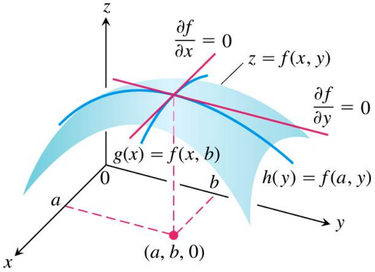

If a local maximum of f occurs at $x = a, y = b$, then the first partial derivatives $f_x(a, b)$ and $f_y(a, b)$ are both zero.

First Derivative Test: If $f(x, y)$ has either a relative maximum or minimum at at point (a, b) then

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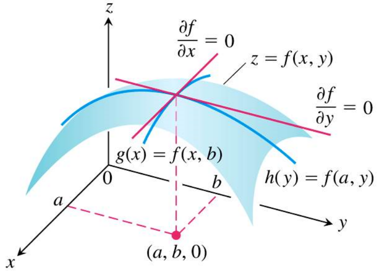

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In other words $\nabla f(a, b) = 0$.

Finding Maxima and Minima on a region

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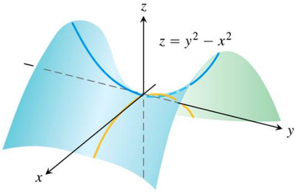
If (a, b) is a point such that $\frac{\partial f}{\partial x}(a, b) = 0$ and $\frac{\partial f}{\partial y}(a, b) = 0$ let

$$D(a, b) \equiv \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - (\frac{\partial^2 f}{\partial x \partial y})^2.$$

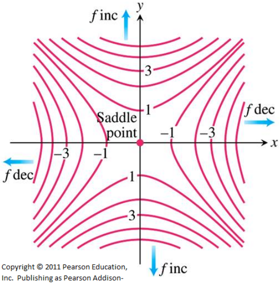
(D is called the discriminant.) Then

- If $D(a, b) > 0$ and $\frac{\partial^2 f}{\partial x^2} > 0$ then (a, b) is a relative minimum.
- If $D(a, b) > 0$ and $\frac{\partial^2 f}{\partial x^2} < 0$ then (a, b) is a relative maximum.
- If $D(a, b) < 0$ then (a, b) is a saddle point (hence neither a relative Max nor a relative min).
- If $D(a, b) = 0$ then we get no information.

Saddle point



(a)



(b)

Problem:

Find all possible relative maxima and minima of

$$f(x, y) = 3x^2 - 6xy + y^3 - 9y$$

and determine the nature of each point.