

## Lecture 13: Sets and counting techniques

- Definitions: Set, subset, empty and universal set.
- Operations with sets: union, intersection, complement, difference.
- Venn diagrams, **Inclusion-Exclusion principle**:  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .
- **Multiplication principle**: Suppose an event  $E$  can occur in  $m$  ways and, independently of this event, an event  $F$  can occur in  $n$  ways. Then, the combination of events  $E$  and  $F$  can occur in  $mn$  ways. (analogously for more than two events)
- **Permutations**: Consider  $n$  objects. A permutation of the  $n$  objects taking  $r$  at a time is any arrangement of any  $r \leq n$  of these objects in a given order. The number of permutations of  $n$  objects taking  $r$  at a time is denoted by  $P(n, r)$ . Using the multiplication principle one can show that  $P(n, r) = \frac{n!}{(n-r)!}$ .

Some examples/exercises:

1. Of the 500 students in some college, 400 are taking a math course, 300 are taking an economics course, and 250 are taking both a math course and an economics course. How many are taking neither a math course nor an economics one?
2. Suppose a restaurant has 3 different appetizers and 2 different entrees. How many ways can you order an appetizer and an entree?
3. A gambler plans to fly from Philadelphia to Las Vegas on Monday, lose his money on Tuesday, and fly back to Philadelphia on Wednesday. There are 4 flights from Philadelphia to Las Vegas on Monday and 5 from Las Vegas to Philadelphia on Wednesday. How many choices of flight bookings are there?
4. Using the multiplication principle, find how many ways you can order the letters  $a, b, c, d, e, f$ .
5. We flip a coin 5 times and write down the results. How many possible outcomes are there?
6. Find the number of three-letter words that can be formed with the letters  $a, b, c, d, e$ .
7. Consider a club with 5 members. How many ways can they choose a president and a vice president?
8. How many five-letter words can be formed with the letters from the word *BABBY*?
9. A code consists of 2 letters followed by 3 digits. Find:
  - (a) Number of codes
  - (b) Number of codes with distinct letters
  - (c) Number of codes with the same letters
10. A classroom has 40 chairs in 4 rows with 10 seats in each row.
  - (a) How many ways can 35 students seat themselves?
  - (b) How many ways can they sit if five particular students must sit in the front row?