HOMEWORK ASSIGNMENT 4

Name: **Due:** Wednesday September 25 (before recitation)

Note: Homework must be submitted online on Canvas (scanned).

Problem 1:

Find the gradient of f, ∇f, at the given point:

1. $f(x, y, z) = x^2 + y^2 - 2z^2 + z \ln x$ at $(1, 1, 1)$
2. $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2} + \ln(xyz)$ at $(-1, 2 - 2)$

Problem 2:

Find the directions in which the functions increase and decrease most rapidly at P_0. Then find the derivatives of the functions in these directions:

1. $f(x, y) = x^2 y + e^{xy} \sin y$ at $P_0(1, 0)$
2. $f(x, y, z) = \ln(xy) + \ln(yz) + \ln(xz)$ at $P_0(1, 1, 1)$.

Problem 3:

Consider $f(x, y) = xy$. Sketch the curve $f(x, y) = -4$ together with ∇f and the tangent line at the point $(2, -2)$. Then write an equation for the tangent line and an equation for the normal line at that point.

Problem 4:

1. In what direction is the derivative of $f(x, y) = xy + y^2$ at $P(3, 2)$ equal to zero?
2. Is there a direction $\vec{u}$ in which the rate of change of $f(x, y) = x^2 - 3xy + 4y^2$ at $P(1, 2)$ equals 14? Give reasons for your answer.

Problem 5:

Find an equation for the tangent plane and an equation for the normal line to the surface $2z - x^2 = 0$ at $P_0(2, 0, 2)$.

Problem 6:

By about how much will

$$g(x, y, z) = e^x \cos(yz)$$

change as the point $P(x, y, z)$ moves from the origin a distance of $ds = 0.1$ in the direction of $2\vec{i} + 2\vec{j} - 2\vec{k}$?

Problem 7:

Find the linearization $L(x, y)$ of the function at each point:

1. $f(x, y) = e^x \cos y$ at $(0, 0)$ and at $(0, \pi/2)$.
2. $f(x, y) = (x + y + 2)^2$ at $(0, 0)$ and at $(1, 2)$.
3. $f(x, y) = x^2 - 3xy + 5$ at $P_0(2, 1)$.

Problem 8:

You plan to calculate the area of a long, thin rectangle from measurements of its length and width. Which dimension should you measure more carefully to obtain a more accurate value of the area? Give reasons for your answer.

Problem 9:

Find all the local maxima, local minima and saddle points of $f(x, y) = x^3 + 3xy^2 - 15x + y^3 - 15y$.

Problem 10:

Read Sections 14.7 and 14.8 of *Thomas' Calculus Early Transcendentals* book.

Midterm 1 covers sections 12.1, 12.2, 12.3, 12.5 and 14.1 to 14.8. Which topics would you like to review most for Midterm 1?