# HOMEWORK ASSIGNMENT 3

**Name:** **Due:** Wednesday September 18 (before recitation)

Note: Homework must be submitted online on Canvas (scanned).

## Problem 1:

Let $w = f(x, y, z)$. Write the formal definition of the partial derivative with respect to $z$, $\partial f/\partial z$, at $(x_0, y_0, z_0)$. *Use this definition* to find $\partial f/\partial z$ at $(1, 2, 3)$ for $f(x, y, z) = x^2yz^2$.

## Problem 2:

The one-dimensional *heat equation*

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$$

describes the evolution of the temperature $f(x, t)$ in a region. Show that $u(x, t) = \sin(\alpha x)e^{-\beta t}$ satisfies the heat equation for some constants $\alpha$ and $\beta$, and find the relationship that these two constant must satisfy.

## Problem 3:

Consider the functions $f(x, y) = 4e^x \ln y$, $x(u, v) = \ln(u \cos v)$, $y(u, v) = u \sin v$. If $w(u, v) = f(x(u, v), y(u, v))$, find $\partial w/\partial u$, $\partial w/\partial v$ in two different ways:

1. By express $w$ explicitly as a function of $u$ and $v$, and then taking partial derivatives.
2. By using the chain rule.

## Problem 4:

Find the value of $\partial z/\partial x$ and $\partial z/\partial y$ at the point $(\pi, \pi, \pi)$ if the equation

$$\sin(x + y) + \sin(y + z) + \sin(x + z) = 0$$

defines $z$ as a function of the two independent variables $x$ and $y$.

## Problem 5:

Show that if $w = f(u, v)$ satisfies the Laplace equation $f_{uu} + f_{vv} = 0$ and if $u = (x^2 - y^2)/2$ and $v = xy$, then $w$ satisfies the Laplace equation $w_{xx} + w_{yy} = 0$.

## Problem 6:

Under mild continuity restrictions, it is true that if

$$F(x) = \int_a^b g(t,x)dt,$$

with $a, b$ some constants, then $F'(x) = \int_a^b g_x(t,x)dt$. Using this fact and the Chain Rule, one can find the derivative of

$$F(x) = \int_a^{f(x)} g(t,x)dt$$

by letting

$$G(u,x) = \int_a^u g(t,x)dt,$$

where $u = f(x)$. Find the derivative of $F(x) = \int_0^{x^2} \sqrt{t^4 + x^3}dt$.

## Problem 7:

Suppose that $T = T(x,y)$ describes the temperature at points $(x,y)$ of a plane and that

$$\frac{\partial f}{\partial x}(x,y) = y, \qquad \frac{\partial f}{\partial y}(x,y) = x.$$

1. Locate the maximum and minimum temperatures on the ellipse whose parametric equations are

$$x(t) = 2\sqrt{2}\cos t, \qquad y(t) = \sqrt{2}\sin t, \qquad 0 \leq t \leq 2\pi.$$

2. If $T(x,y) = xy - 2$, find the maximum and minimum values of $T$ on the ellipse.

## Problem 8:

Find the gradient of the function at the given point. Then sketch the gradient together with the level curve that passes through the point:

1. $f(x,y) = xy^2, \quad (2,-1),$
2. $f(x,y) = \ln(x^2 + y^2), \quad (1,1).$

## Problem 9:

Find the derivative of the function $f(x,y) = \dfrac{x-y}{xy+2}$ at $P_0(1,-1)$ in the direction of $\vec{u} = 12\vec{i} + 5\vec{j}$.

## Problem 10:

Read Sections 14.6 and 14.7 of *Thomas' Calculus Early Transcendentals* book.