

## HOMEWORK ASSIGNMENT 8

Name:

Due: Friday April 19, 4pm

### PROBLEM 1:

Compute  $A^T A$  and  $AA^T$  and their eigenvalues and unit eigenvectors for  $V$  and  $U$ .

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

Check  $AV = U\Sigma$  (this decides  $\pm$  signs in  $U$ ).  $\Sigma$  has the same shape as  $A$ : 2 by 3.

### PROBLEM 2:

Suppose  $A$  has orthogonal columns  $\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n$  of lengths  $\sigma_1, \sigma_2, \dots, \sigma_n$ . What are  $U$ ,  $\Sigma$  and  $V$  in the SVD?

### PROBLEM 3:

Compute  $A^T A$  and  $AA^T$  and their eigenvalues and unit eigenvectors when the matrix is  $A = \begin{bmatrix} 3 & 4 & 0 \end{bmatrix}$ . What are the singular values of  $A$ ?

### PROBLEM 4:

Calculate the singular value decomposition of  $A$ :

$$A = \begin{bmatrix} 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}^T.$$

Write out the pseudoinverse  $V\Sigma^+U^T$  of  $A$ . Compute  $AA^+$  and  $A^+A$ :

### PROBLEM 5:

Find  $A^+$  and  $A^+A$  and  $AA^+$  and  $\mathbf{x}^+$  (shortest length least square solution) for this matrix  $A = U\Sigma V^T$  (the SVD is given below) and these  $\mathbf{b}$ :

$$A = \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} .6 & -.8 \\ .8 & .6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \quad \mathbf{b}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \text{ and } \mathbf{b}_2 = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

### PROBLEM 6: CHALLENGE PROBLEMS FROM THE ZYBOOK

Challenge activity 8.1.1 of the zyBook. This is not optional.

### PROBLEM 7:

Read Section 9.1 from the zyBook (Markov chains) and do all of the participation exercises therein. Which concept was most confusing for you?