Name: ______________________________________ **PennID**: _________________

Math 425/AMCS 525
Midterm 2 (Practice Exam)
March 24, 2019

Please *turn off and put away all electronic devices.* You are allowed to use an index card (3x5") with hand-written notes on both sides during this exam. No calculators, no books. Read the problems carefully. **Show all work** (answers without proper justification will not receive full credit). Be as organized as possible: illegible work will not be graded.

Please sign and date the pledge below to comply with the Code of Academic Integrity. Don't forget to write your Name and PennID on the top of this page. Good luck!

#	Points possible	Your score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

____ _____________________

Signature Date

Problem 1 (20 pts)

Solve the following problem

$$\begin{aligned}
&\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + u = 0, \qquad -\infty < x < \infty,\ t > 0,\\
&u(x,0) = e^{2x}.
\end{aligned}$$

Simplify the formula for $u(x,t)$ as much as you can.
(*Hint:* let $u(x,t) = v(x,t)e^{-t}$.)

Solution (Problem 1):

Solution (Problem 1):

Problem 2 (20 pts)

Solve the inhomogeneous diffusion problem on the half-line

$$\begin{aligned} &v_t - kv_{xx} = f(x,t) \qquad && \text{for } 0 < x < \infty,\ 0 < t < \infty, \\ &v(0,t) = h(t), && v(x,0) = \phi(x). \end{aligned}$$

Solution (Problem 2):

Solution (Problem 2):

Problem 3 (20 pts)

Solve the equation

$$\frac{\partial^2 u}{\partial t^2} = 2\frac{\partial^2 u}{\partial x^2} + u, \quad \text{for } 0 < x < \pi,\ t > 0,$$

subject to the following boundary and initial conditions:

$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0$$
$$u(x,0) = 1 + 3\cos(x), \quad \frac{\partial u}{\partial t}(x,0) = 1 + 7\cos(2x).$$

Solution (Problem 3):

Solution (Problem 3):

Problem 4 (20 pts)

Consider the following eigenvalue problem

$$\begin{aligned} &X''(x) = -\lambda X(x), \quad 0 < x < 1, \\ &X(0) = 0, \\ &3X(1) - X'(1) = 0. \end{aligned}$$

(a) Prove that all eigenvalues are real.

(b) How many positive eigenvalues are there? Explain your answer.

(a) How many negative eigenvalues are there? Explain your answer.

Solution (Problem 4):

Solution (Problem 4):

Problem 5 (20 pts)

Let

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & 1 < x \leq 2. \end{cases}$$

(a) Find the Fourier sine series of the function $f(x)$ on $[0, 2]$.

(b) For each $0 \leq x \leq 2$, what is the sum of the series?

(c) Does the series converge uniformly to $f(x)$ on $[0, 2]$? Why?

(d) Does the series converge in the L^2 sense to $f(x)$ on $(0, 2)$? Why?

Solution (Problem 5):

Solution (Problem 5):

Extra paper

Extra paper

Extra paper

Extra paper