

HOMWORK ASSIGNMENT 6

Name:

Due: Wednesday March 27

PROBLEM 1:

A 3 by 3 matrix B is known to have eigenvalues 0,1,2. Using this information, find:

1. The rank of B .
2. The determinant of $B^T B$.
3. The eigenvalues of $(B^2 + I)^{-1}$.

PROBLEM 2:

Suppose A has eigenvalues 0, 3, 5 with independent eigenvectors \mathbf{u} , \mathbf{v} , \mathbf{w} .

1. Give a basis for the nullspace and a basis for the column space.
2. Find a particular solution to $A\mathbf{x} = \mathbf{v} + \mathbf{w}$. Find all solutions.
3. $A\mathbf{x} = \mathbf{u}$ has no solution. If it did then ___ would be in the column space.

PROBLEM 3:

Diagonalize the matrix

$$A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 4 & 0 \\ 5 & 0 & 0 \end{bmatrix}.$$

What are its eigenvectors and eigenvalues?

PROBLEM 4:

True or false: If the eigenvalues of a 3 by 3 matrix A are 2, 2, 5 then the matrix is certainly

1. invertible
2. diagonalizable
3. not diagonalizable

PROBLEM 5:

1. $A^k = X\Lambda^k X^{-1}$ approaches the zero matrix as $k \rightarrow \infty$ if and only if every λ has absolute value less than _____. Which of these matrices has $A^k \rightarrow 0$?

$$A_1 = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} .6 & .9 \\ .1 & .6 \end{bmatrix}.$$

2. Find Λ and X to diagonalize A_1 . What is the limit of Λ^k as $k \rightarrow \infty$? What is the limit of $X\Lambda^k X^{-1}$? In the columns of this limiting matrix you see the _____.

PROBLEM 6:

Suppose $A\mathbf{x} = \lambda\mathbf{x}$. If $\lambda = 0$ then \mathbf{x} is in the nullspace. If $\lambda \neq 0$ then \mathbf{x} is in the column space. Those spaces have dimensions $(n-r)+r = n$. So why doesn't every square matrix have n linearly independent eigenvectors?

PROBLEM 7: CHALLENGE PROBLEMS FROM THE ZYBOOK

Challenge activities in 7.2.1 and 7.3.1 of the zyBook. These are not optional.

PROBLEM 8:

Study the material for the second midterm exam (Lecture Notes 10 to 18 on Canvas; correspondence in zyBook: Chapters 4, 5, 6 and Sections 7.1, 7.2, 7.3, 7.5). Which concept would you most like to review?