HOMEWORK ASSIGNMENT 3

Name: **Due:** Monday February 11, 1PM

All the problems in this homework are from W. Strauss book. The level of difficulty is marked as *,**,***.

Problem 1*: Strauss, Section 2.1 #2, p.38

Solve $u_{tt} = c^2 u_{xx}$, $u(x,0) = \log(1+x^2)$, $u_t(x,0) = 4 + x$.

Problem 2*: Strauss, Section 2.1 #3, p.38

The midpoint of a piano string of tension T, density ρ, and length l is hit by a hammer whose head diameter is $2a$. A flea is sitting at a distance $l/4$ from one end (assume that $a < l/4$; otherwise, poor flea!). How long does it take for the disturbance to reach the flea?

Problem 3*: Strauss, Section 2.1 #7, p.38

If both ϕ and ψ are odd functions of x, show that the solution $u(x,t)$ of the wave equation is also odd in x for all t.

Problem 4**: Strauss, Section 2.1 #10, p.38

Solve $u_{xx} + u_{xt} - 20u_{tt} = 0$, $u(x,0) = \phi(x)$, $u_t(x,0) = \psi(x)$.

Problem 5**: Strauss, Section 2.1 #11, p.38

Find the general solution of $3u_{tt} + 10u_{xt} + 3u_{xx} = \sin(x+t)$.

Problem 6*: Strauss, Section 2.2 #1, p.41

Use the energy conservation of the wave equation to prove that the only solution with $\phi \equiv 0$ and $\psi \equiv 0$ is $u \equiv 0$. (Hint: Use the first vanishing theorem in Section A.1.)

Problem 7**: Strauss, Section 2.2 #5, p.41

For the *damped* string (equation below), show that the energy (defined as for the usual wave equation) decreases:

$$u_{tt} - c^2 u_{xx} + r u_t = 0, \qquad r > 0.$$

(Hint: Two possible ways: 1) Proceed as in class, 2) Multiply the equation by u_t, identify the derivative of a square and integrate by parts).

Problem 8**: Strauss, Section 2.3 #4, p.46

Consider the diffusion equation $u_t = u_{xx}$ in $\{0 < x < 1, 0 < t < \infty\}$ with $u(0,t) = u(1,t) = 0$ and $u(x,0) = 4x(1-x)$.

a) Show that $0 < u(x,t) < 1$ for all $t > 0$ and $0 < x < 1$.

b) Show that $u(x,t) = u(1-x,t)$ for all $t \geq 0$ and $0 \leq x \leq 1$.

c) Use the energy method to show that $\int_0^1 u^2(x,t)dx$ is a strictly decreasing function of t.

Problem 9**: Strauss, Section 2.3 #6, p.46

Prove the *comparison principle* for the diffusion equation: If u and v are two solutions, and if $u \leq v$ for $t = 0$, for $x = 0$ and for $x = l$, then $u \leq v$ for $0 \leq x \leq l$, $0 \leq t < \infty$.

Problem 10**: Strauss, Section 2.4 #15, p.53

Prove the uniqueness of the diffusion problem with Neumann boundary conditions by the energy method:

$$u_t - u_{xx} = f(x,t) \text{ for } 0 < x < l, t > 0, u(x,0) = \phi(x), u_x(0,t) = g(t), u_x(l,t) = h(t).$$

Problem 11**: Strauss, Section 2.4 #16, p.54

Solve the diffusion equation with constant dissipation:

$$u_t - u_{xx} + bu = 0 \text{ for } -\infty < x < \infty, \text{ with } u(x,0) = \phi(x),$$

where $b > 0$ is a constant. (Hint: Make the change of variables $u(x,t) = e^{-bt}v(x,t)$.)

Problem 12:

Read sections 2.3, 2.4 and 2.5 of W. Strauss book. Which concept was most confusing to you?
Which part so far would you like to review most before the exam (lecture of Tuesday February 12)?