True or False

1. Each one of the matrices A^TA and AA^T is symmetric.

2. Let A, B, C be invertible matrices. Then, the product ABC is always invertible.

3. If A and B have inverses A^{-1} and B^{-1}, then $(A+B)^{-1} = A^{-1} + B^{-1}$.

4. If AB is invertible, then A and B are invertible.

5. If an n by n matrix B is invertible, then the product AB is invertible for any m by n matrix A.

6. Let M an n by n matrix such that $M\vec{x} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ x_n - x_1 \end{bmatrix}$ for all $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$. Then, rank($M$)$= n-1$.

7. If A and B are matrices with rank 1, then so is $(A+B)/2$.

8. If a 3x4 matrix has a RREF with only three pivots, then its rows are linearly dependent.

9. If a 3x4 matrix has a RREF with three pivots, then its columns must span $\mathbb{R}^3$.

10. If a 4x3 matrix has 3 pivots, then $A\vec{x} = \vec{b}$ always has at least a solutions

11. If a 5x3 matrix has 3 pivots, then $A\vec{x} = \vec{b}$ always has a solution.

12. If a 3x5 matrix has 3 pivots, then $A\vec{x} = \vec{b}$ always has infinite solutions.

13. If a 3x5 matrix has RREF with three pivots, then its rows form a basis of $\mathbb{R}^5$.

14. If a 5x3 matrix A has RREF with three pivots, then the columns of A are linearly independent.

15. If P_{12} is the matrix that changes row 1 with row 2, then $P_{12}^2 = I$ (I is the identity matrix).

16. Let E be the elimination matrix that adds 2 times row 1 to row 2. Then E^{-1} adds 1/2 times row 1 to row 2.

17. Consider a matrix that adds row1 to row 2 and at the same time adds row 2 to row 1. Then, the inverse matrix subtracts row 1 from row 2 and at the same time subtracts row 2 from row 1.

18. Let $A = LU$ (L lower triangular with ones on the diagonal, U upper triangular). Let $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$ be the columns of A and $\vec{u}_1$, $\vec{a}_2$, $\vec{a}_3$ be the columns of U. Then, if $\vec{u}_3 = 2\vec{u}_2 - \vec{u}_1$, we also have that $\vec{a}_3 = 2\vec{a}_2 - \vec{a}_1$.

19. If a vector $\vec{k}$ lies in the nullspace of A^T and if $A\vec{x} = \vec{b}$, then $A(\vec{x} + \vec{k})$ also equals $\vec{b}$.

20. An square matrix A such that $N(A) = \{0\}$ always has an inverse.

21. The matrices A and $-A$ have the same four subspaces.

22. The matrices A and A^T have the same number of pivots.

23. Let A be a square matrix such that $A\vec{x} = \vec{0}$ for some $\vec{x} \neq \vec{0}$. Then A can never have an inverse.

24. The set consisting of the x-axis, the y-axis, the line $y = x$ and the line $y = -x$ forms a subspace of $\mathbb{R}^2$.

25. The set of points satisfying both $y = 3x + z$ and $z + x = 0$ is a subspace of $\mathbb{R}^3$.

26. The set of points satisfying $x^2 + y = 0$ is a subspace of $\mathbb{R}^2$.

27. The set of points satisfying $x^2 + y^2 = 0$ is a subspace of $\mathbb{R}^2$.

28. The set of points satisfying both $y = 3x + z$ and $z + x = 2$ is a subspace of $\mathbb{R}^3$.

29. Let V be the set of 2 by 2 matrices. Then, the set of symmetric 2 by 2 matrices is a subspace of V.

30. Consider the set of vectors $\vec{x}$ such that $A\vec{y} = \vec{x}$ always has a solution. Is that a subspace?

31. The set of points satisfying $y = 3x^2$ is a subspace of $\mathbb{R}^2$.

32. Let V be the vector space of polynomials with degree no more than 3. The set of polynomials of the form $p(x) = ax^2$ with $a \in \mathbb{R}$ is a subspace of V.

33. Given a basis, one can always find an orthonormal one with the same span.

34. For any basis $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ of the vector space U, it holds that $\vec{u} = (\vec{u}_1 \cdot \vec{u})\vec{u}_1 + (\vec{u}_2 \cdot \vec{u})\vec{u}_2 + (\vec{u}_3 \cdot \vec{u})\vec{u}_3$, for all $\vec{u} \in U$.

35. In $\mathbb{R}^3$, the x-axis and z-axis are orthogonal complements.

36. The set of polynomials of degree exactly 3 is a vector space.

37. In $\mathbb{R}^2$, the orthogonal complement of a line is another line.

38. In $\mathbb{R}^3$, the orthogonal complement of a line is another line.

39. The line spanned by $(1, 1, 1)$ is the orthogonal complement of $x + y + z = 0$ in $\mathbb{R}^3$.

40. In $\mathbb{R}^3$, the orthogonal complement of a line is a plane.

41. In $\mathbb{R}^3$, the orthogonal complement of a plane is another plane.

42. In $\mathbb{R}^3$, the orthogonal complement of a line is a vector space with dimension equal to 2.

43. If A is a 3 by 4 matrix with rank(A)=2, then the nullspace of A is a line.

44. If A is a 5 by 4 matrix with rank(A)=3, then the orthogonal complement of the row space of A has dimension 2.

45. If A is invertible, then $A(A^TA)^{-1}A^T = I$ always hold (I is the identity matrix).

46. In $\mathbb{R}^3$, the projection matrix onto a plane has rank 1.

47. The matrix that projects onto the column space of A is $A(A^TA)^{-1}A^T$.

48. In $\mathbb{R}^3$, the projection matrix onto a line has rank 1.

49. In $\mathbb{R}^3$, the nullspace of the projection matrix onto a line is another line perpendicular to it.

50. In $\mathbb{R}^3$, the rank of the projection matrix onto a line is 2.

51. If $\det(A) = -1$ for some matrix A, then there is some $\vec{b}$ for which $A\vec{x} = \vec{b}$ has infinitely many solutions.

52. For any square matrices A, $\det(2A) = 2\det(A)$.

53. For any square matrices A and B, $\det(A + B) = \det(A) + \det(B)$.

54. If A is a 2 by 2 matrix with eigenvalues -1 and 2, and B is a 2 by 2 matrix with eigenvalues 0 and 2, then $\det((B - I)^2A^{-1}) = 1$.

55. If A is a 3x3 matrix with determinant 1, then $2A$ has determinant 6.

56. If A is a 2 by 2 matrix with eigenvalues -1 and 2, and B is a 2 by 2 matrix with eigenvalues 0 and 1, then $\det((B + I)A^{-1}) = 1$.

57. If a square matrix A has 4 as an eigenvalue, then $A-3I$ must have 1 as an eigenvalue.

58. Let A be an n by n matrix. If n is odd and A is skew-symmetric (i.e., $A^T = -A$), then A is not invertible.

59. The eigenvalues of 2A are 2 times the eigenvalues of A.

60. The eigenvalues of A and A^T are the same.

61. The eigenvalues and eigenvectors of A and A^T are the same.

62. If $\det(A^2) = 1$, then A's eigenvalues must all be 1 or -1.

63. If $A^3 = 0$ for some square matrix A, then all the eigenvalues of A are zero.

64. If $\vec{u}_1$ and $\vec{u}_2$ are eigenvectors of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 7 \end{bmatrix}$ corresponding to distinct eigenvalues, then $\vec{u}_1^T \vec{u}_2 = 0$.

65. If A is symmetric, then so is e^A.

66. If A is singular, then so is e^A.

67. A 3 by 3 symmetric matrix with eigenvalues $0, 0, 1$ always has rank(A)=1.

68. If A is a square matrix and B is obtained from A via row operation R2'=R2+3R1, then B has the same eigenvalues as A.

69. For any matrix A, the eigenvectors corresponding to distinct eigenvalues are perpendicular.

70. If A is invertible and has one eigenvalue λ, then $1/\lambda$ is an eigenvalue of A^{-1}.

71. A basis for eigenvectors for nonzero eigenvalues of A is a basis for $C(A)$ for any matrix A.

72. The eigenvectors for the zero eigenvalue are the null space of A.

73. The only upper triangular 3×3 matrix with 1s on the diagonal which is diagonalizable is the identity matrix.

74. The matrix $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$ has an eigenvalue equal to 9.

75. Let P be the matrix which projects vector of $\mathbb{R}^3$ onto the plane $x + y + z = 0$. The eigenvalues of P are 0,1,1.

76. Let P be a matrix which projects vector of $\mathbb{R}^3$ onto a line. The eigenvalues of P are 1,0,0.

77. If A is a rotation matrix, then it cannot have real eigenvalues.

78. A projection matrix can only have eigenvalues equal to 1 or 0.

79. A 2 by 2 matrix that rotates every vector 90^{o} cannot have any real eigenvalues.

80. If a 3 by 3 matrix has all three eigenvalues different than zero, then the eigenvectors form a basis of $\mathbb{R}^3$.

81. A 3 by 3 matrix with eigenvalues $0, 1, -1$ is always diagonalizable.

82. A singular matrix is never diagonalizable.

83. If A and B are diagonalizable, then so is $A + B$.

84. The transformation $T : \mathbb{R}^2 \to \mathbb{R}$ given by $T(\vec{x}) = x_1^2 + x_2$ is linear (here $\vec{x} = (x_1, x_2)$).

85. The transformation $T : \mathbb{R}^2 \to \mathbb{R}$ given by $T(\vec{x}) = 1 - x_1^2 - x_2$ is linear (here $\vec{x} = (x_1, x_2)$).

86. The transformation $T : \mathbb{R}^2 \to \mathbb{R}$ given by $T(\vec{x}) = x_1 - x_2$ is linear (here $\vec{x} = (x_1, x_2)$).

87. If A is a change of basis matrix from a basis v_i to a basis u_i, and B is a change of basis matrix from the basis u_i to a basis w_i, then AB is a change of basis matrix from v_i to w_i.

88. Consider a plane in $\mathbb{R}^3$ and a basis U for it. The projection of vectors in $\mathbb{R}^3$ onto that plane is a linear transformation. The corresponding matrix is 2 by 2 when we use U as output basis.

89. Is the following transformation linear? $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(\vec{x}_0) =$ solution to the system given by $\vec{x}' = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \vec{x}$ with initial condition $\vec{x}_0$.

90. Is the following transformation linear? $\begin{aligned} T &: \mathbb{R}^3 \to \mathbb{R}^3 \\ \vec{x} &\to T(\vec{x}) = \min_{i=1,2,3} x_i \end{aligned}$.

91. Any matrix with orthonormal columns always has orthonormal rows.

92. An orthogonal matrix always preserves lengths and angles.

93. If Q is an orthogonal matrix, then the corresponding linear transformation preserves lenghts and angles, i.e., length of $Q\vec{x}$ is equal to length of $\vec{x}$ and the angle between $\vec{x}$ and $\vec{y}$ is equal to the angle between $Q\vec{x}$ and $Q\vec{y}$.

94. An orthogonal matrix always has eigenvalues with modulo 1.

95. A symmetric and orthogonal matrix always has eigenvalues equal to 1 or -1.

96. If A is an orthogonal matrix, then $\lambda = 2$ cannot be an eigenvalue.

97. For any M symmetric matrix, it is impossible to find two eigenvectors $\vec{u}_1 \neq \vec{u}_2$ such that $\vec{u}_1^T \vec{u}_2 \neq 0$.

98. It is possible to find a real matrix A such that the matrix AA^T have $\lambda = -1$ as an eigenvalue.

99. It is possible to find a real matrix A such that the matrix AA^T have $\lambda = 0$ as an eigenvalue.

100. A definite positive matrix always has an inverse.

101. The matrix $A^T A$ is always positive semidefinite.

102. Let A be a 2 by 5 matrix such that AA^T has eigenvalues 1 and 2. Then $\dim(N(A^T A)) = 2$.

103. For any matrix A, the nullspace of A is exactly the same as the nullspace of $A^T A$.

104. For any orthogonal matrix Q, all its singular values are 1.

105. Let $\vec{x} \neq \vec{0}$ be a vector in $\mathbb{R}^3$. Then the matrix $A = \vec{x}\vec{x}^T$ has eigenvalues $\lambda_1 = 0 = \lambda_2$, $\lambda_3 > 0$.

106. If $m < n$, then $A^T A$ cannot be positive definite.

107. If A and B are positive definite, then so is $(A + B)/2$.

108. If a square matrix A is stochastic (nonnegative entries with columns adding to 1), then so is e^A.

109. If square matrices A and B are stochastic (nonnegative entries with columns adding to 1), then so is $(A + B)/2$.

110. A real matrix A has eigenvalues $\lambda_1 = -3$, $\lambda_2 = 1$, $\lambda_3 = 3$. Then A has at least a non positive entry.

111. A real matrix A has eigenvalues $\lambda_1 = 4$, $\lambda_2 = -3$, $\lambda_3 = 1 + i$, $\lambda_4 = 1 - i$. Then all entries of A have to be positive.

112. A real square matrix with positive entries cannot have negative eigenvalues.

113. A real square matrix with positive entries always has $\lambda = 1$ as an eigenvalue, and the modulo of all the other eigenvalues is less than 1.

114. A real square matrix whose columns add up to 1 always has $\lambda = 1$ as an eigenvalue.

115. A real square matrix whose columns add up to 1 always has $\lambda = 1$ as an eigenvalue and the modulo of all the other eigenvalues is less than 1.

116. Let A be a 3 by 3 Markov matrix. Then $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A^T.

117. A real matrix A has eigenvalues $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = -2$, $\lambda_4 = 0$. Then A cannot have all of its entries positive.

118. A real matrix A has eigenvalues $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = 2$, $\lambda_4 = 0$. Then A cannot have all of its entries positive.

119. A real matrix A has eigenvalues $\lambda_1 = 1/2$, $\lambda_2 = 1$, $\lambda_3 = 1$, $\lambda_4 = -1/2$. Then A cannot have all of its entries positive.

120. Let A be a square real matrix with positive entries. It is possible for A to have the following eigenvalues: $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = 2$, $\lambda_4 = 0$.

121. A real matrix A has eigenvalues $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = 1/2$, $\lambda_4 = 0$. We also know that $(1, 1, 1, 1)$ is an eigenvector of A^T corresponding to $\lambda = 1$. Then we know that A correspond to a Markov chain and that this has a unique steady state.

122. We know that the maximum positive eigenvalue of a real matrix A is $\lambda = 3$ with eigenvector $\vec{u} = \begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$. Then we can ensure that A have at least one non positive entry.

123. We know that a real matrix A has eigenvalues $\lambda_1 = 3$, $\lambda_2 = 1$, $\lambda_2 = -2$ with first eigenvector $\vec{u}_1 = \begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$. Then we can ensure that A has at least one non positive entry.

124. If A is a real square matrix whose columns add up to one, then the entries of $A\vec{u}$ sum up to the same quantity as the entries of $\vec{u}$.

125. If a square matrix A has all its entries strictly negative, then it must have a strictly negative eigenvalue, and one can choose a strictly negative eigenvector corresponding to that eigenvalue.

126. The incidence matrix of a directed graph is always symmetric.

127. The incidence matrix of a directed graph is always square.

128. The $n \times 1$ vector $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ is an eigenvector of any $m \times n$ incidence matrix.

129. The dimension of the null space of an incidence matrix is never smaller than the dimension of the left null space of an incidence matrix.

130. The adjacency matrix of an undirected graph is always symmetric.

131. The adjacency matrix of an undirected graph is always square.

132. If a graph G is connected, then every diagonal entry of A^2 is nonzero for the adjacency matrix A.

133. If the standard form of an LP has $\vec{b}$ with all nonnegative entries, then the origin $(x_1 = ... = x_n = 0)$ is in the fundamental domain.

134. If the standard form of an LP has $\vec{b} \geq 0$ and $\vec{c} \leq \vec{0}$, then the maximum is attained at the origin.

135. If A is a 5×3 matrix, and $A = U\Sigma V^T$ is a singular value decomposition, and Σ has two nonzero entries, then the null space of AA^T has dimension 3.

136. If A is a 5×3 matrix, then any basis of $N(A)$ can be used for the last columns of V in an SVD $A = U\Sigma V^T$