HOMEWORK ASSIGNMENT 9

Name: **Due:** Wednesday November 21 4pm

Problem 1: Strang 7.4 #10, page 399

Compute A^TA and AA^T and their eigenvalues and unit eigenvectors when the matrix is $A = \begin{bmatrix} 3 & 4 & 0 \end{bmatrix}$. What are the singular values of A?

Problem 2: Strang 7.4 #1, page 399

Calculate the singular value decomposition of A:

$$A = \begin{bmatrix} 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}^T.$$

Write out the pseudoinverse $V\Sigma^+U^T$ of A. Compute AA^+ and A^+A:

Problem 3: Strang 7.4 #18, page 400

Find A^+ and A^+A and AA^+ and $\mathbf{x}^+$ (shortest length least square solution) for this matrix $A = U\Sigma V^T$ (the SVD is given below) and these $\mathbf{b}$:

$$A = \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} .6 & -.8 \\ .8 & .6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \qquad \mathbf{b}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \text{ and } \mathbf{b}_2 = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

Problem 4:

We are given six centered data points in $\mathbb{R}^3$:

$$X = \begin{bmatrix} \mathbf{x}_1 & \dots & \mathbf{x}_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & 2 & 1 & 0 & -3 & -1 \\ 1 & 1 & 0 & -1 & 2 & -3 \end{bmatrix}.$$

The SVD of X is

$$X \approx \begin{bmatrix} .44 & 0 & -.90 \\ .83 & -.39 & .40 \\ .35 & .92 & .17 \end{bmatrix} \begin{bmatrix} 4.44 & & & & & \\ & 4 & & & & \\ & & 0.55 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} .36 & .13 & -.59 & .41 & .06 & .57 \\ .55 & .03 & .14 & 0 & .77 & -.30 \\ .19 & -.10 & .73 & 0 & 0 & .65 \\ -.08 & -.23 & -.31 & -.82 & .26 & .33 \\ -.50 & .76 & .07 & 0 & .38 & .18 \\ -.52 & -.59 & -.03 & .41 & .44 & .10 \end{bmatrix}^T$$

Find the line of best fit through the origin and the projection of each point onto this line. Plot these projections in the basis given by the first principal component of X.

Problem 5: Strang 10.3 #1, page 480

Find the eigenvalues of this Markov matrix (their sum is the trace):

$$A = \begin{bmatrix} 0.90 & 0.15 \\ .10 & 0.85 \end{bmatrix}.$$

What is the steady state eigenvector for the eigenvalue $\lambda_1 = 1$?

Problem 6: Strang 10.3 #5, page 481

Every year 2% of young people become old and 3% of old people become dead. (No births). Find the steady state for

$$\begin{bmatrix} \text{young} \\ \text{old} \\ \text{dead} \end{bmatrix}_{k+1} = \begin{bmatrix} .98 & .00 & 0 \\ .02 & .97 & 0 \\ .00 & .03 & 1 \end{bmatrix} \begin{bmatrix} \text{young} \\ \text{old} \\ \text{dead} \end{bmatrix}_{k}.$$

Problem 7: Strang 10.3 #9, page 481

Prove that the square of a Markov matrix is also a Markov matrix.

Problem 8: Strang 10.3 #11, page 481

Complete A to a Markov matrix and find the steady state eigenvector. When A is a symmetric Markov matrix, why is $\mathbf{x}_1 = (1, \ldots, 1)$ its steady state?

$$A = \begin{bmatrix} .7 & .1 & .2 \\ .1 & .6 & .3 \\ - & - & - \end{bmatrix}.$$

Problem 9: Strang 10.3 #20, page 482

Suppose $B > A > 0$, meaning that each $b_{ij} > a_{ij} > 0$. How does the Perron–Frobenius discussion show that $\lambda_{\max}(B) > \lambda_{\max}(A)$?

Problem 10:

What type of applications would you most like to see in the rest of the class?