HOMEWORK ASSIGNMENT 11

Name: **Due:** Monday December 10, 4pm

Note about notation: for damping, one convention of the transition matrix with damping uses a number d which is the probability of clicking a link on a given page, so that $1 - d$ is the probability of jumping to a random page. Another convention is to use a number α which is the probability of jumping to a random page, while $1 - \alpha$ is the probability of clicking a link on a given page. You can see that $d = 1 - \alpha$.

Problem 1: review of bases

Under what condition on the numbers $m_1, m_2, ..., m_9$ do these three parabolas give a basis for the space of all parabolas $a + bx + cx^2$?

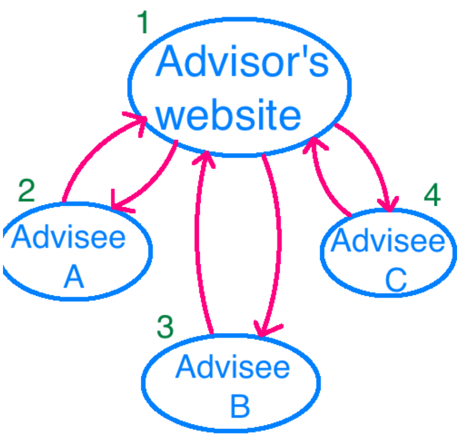
$$\vec{v}_1 = m_1 + m_2 x + m_3 x^2, \quad \vec{v}_2 = m_4 + m_5 x + m_6 x^2, \quad \vec{v}_3 = m_7 + m_8 x + m_9 x^2$$

Problem 2: review of stochastic matrices

Find the eigenvalues and eigenvectors of A. Explain why A^k approaches A^∞.

$$A = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \qquad A^\infty = \begin{bmatrix} .6 & .6 \\ .4 & .4 \end{bmatrix}$$

Problem 3: damping



a) Write the naive transition matrix A for this graph (this would correspond to $d = 1$ or $\alpha = 0$), adhering to the numbering of nodes shown in the image.

b) Calculate the following four vectors:

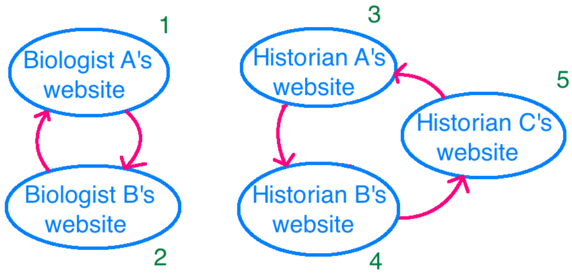
$$A\begin{bmatrix}.25\\.25\\.25\\.25\end{bmatrix},\quad A^2\begin{bmatrix}.25\\.25\\.25\\.25\end{bmatrix},\quad A^3\begin{bmatrix}.25\\.25\\.25\\.25\end{bmatrix},\quad A^4\begin{bmatrix}.25\\.25\\.25\\.25\end{bmatrix}$$

c) Why is there no steady state for the chain $A^k\begin{bmatrix}.25\\.25\\.25\\.25\end{bmatrix}$? Note that you would **not** be able to determine a page rank here by iterating multiplication by A.

d) What is the transition matrix B with damping $d = .85$ (so $\alpha = .15$)? What's the eigenspace of 1 for B?

e) Calculate the page rank with damping using part d).

Problem 3: more damping



a) Write the naive transition matrix A for this graph (this would correspond to $d = 1$), adhering to the numbering of nodes shown in the image.

b) Calculate the eigenspace of 1 for A. Explain how this would lead to multiple different page ranks.

c) Calculate the transition matrix B with $d = .85$ (so $\alpha = .15$), and find the eigenvector with eigenvalue 1 and entries summing to 1. What is the page rank with damping?

Problem 4: Strang 10.4 #2, page 489

Draw the region in the xy plane where $x + 2y \leq 6$, $2x + y \leq 6$, and $x \geq 0, y \geq 0$. It has four corners. Which point in this "feasible set" minimizes the cost $c = 2x - y$?

Problem 5: Nutrition

Suppose that each day, you need to eat at least 70 grams of protein, 3000 calories, 1 gram of calcium, and 12 mg of iron. And suppose you are shopping for the following 5 items, which have cost and nutrients per 100g as shown.

	Protein (g)	Calories	Calcium (g)	Iron (mg)	Cost ($)
Brown bread	12.0	246	0.1	3.2	.5
Cheese	24.9	423	0.2	0.3	2
Butter	0.1	793	0.03	0.0	1
Baked beans	6.0	93	0.05	2.3	.25
Spinach	3.0	26	0.1	2.0	.25

a) Let x_1 be the number of 100g units of brown bread purchased, x_2 the number of 100g units of cheese purchased, x_3 the number of 100g units of butter, x_4 the number of 100g units of baked beans, and x_5 the number of 100g units of spinach. Write the inequalities coming from the constraints on nutrition, and write the cost equation.

Problem 6: duality

Find the maximum value taken by $f(x, y) = 2x + 3y$ for non negative x and y subject to the three constraints $x \leq 4$, $x + y \leq 10$ and $y \leq 6$ using only geometric techniques. More precisely,

1. Dra the feasible region D as a subset of $\mathbb{R}^2$.
2. Identify the five corner points of D: what are the (x, y) values of these corner points?
3. Evaluate f at each of the corner points and find the maximum.

Problem 7: the simplex method

Now we will solve the optimization problem from Problem 6 using the Simplex method.

1. Express the optimization problem in standard form: meaning, find A, $\mathbf{b}$ and $\mathbf{c}$ so that we are being asked to maximize $\mathbf{c}^T \begin{bmatrix} x \\ y \end{bmatrix}$ subject to $A \begin{bmatrix} x \\ y \end{bmatrix} \leq \mathbf{b}$ and $\begin{bmatrix} x \\ y \end{bmatrix} \geq \mathbf{0}$.
2. Write down the augmented block matrix

$$B = \begin{bmatrix} 1 & -\mathbf{c}^T & \mathbf{0} & \vdots & \mathbf{0} \\ 0 & A & Id & \vdots & \mathbf{b} \end{bmatrix}$$

The penultimate three columns correspond to slack variables $r, s, t \geq 0$. At this initial state, these are pivot variables while x and y are not.

3. On B, perform the row operations needed by the simplex algorithm of B. Explain how you are selecting each column and row to produce a new pivot.

Problem 8

Study for the final! It is comprehensive. Which topic would you most like to review?