

The Course So Far

Here are some things you should be able to do in the course so far:

- Can you write the intersection of planes, lines, etc as a matrix equation?
- Do you know how to go back and forth between sets of linear equations and a matrix equation?
- Do you know how to write the linear equation corresponding to a certain property being satisfied? (i.e. “the sum of the coordinates of a vector \vec{v} in \mathbb{R}^3 is zero” corresponds to the equation $v_1 + v_2 + v_3 = 0$)
- Given a matrix A , can you tell when A is invertible? Do you know how to calculate the inverse if it exists? Can you factor it into LU where U is upper triangular and L is invertible?
- Do you know how to write the elimination matrix that corresponds to a row transformation? Do you understand what powers of elimination matrices do? Do you know how to calculate their inverses?
- Given a matrix A , can you figure out for which vectors \vec{b} there is a solution to $A\vec{x} = \vec{b}$? Can you figure out **how many** solutions there are? Do you know how this is related to the above question?
- Given a function f from \mathbb{R}^n to \mathbb{R}^m (i.e. projection, rotation, or something even more complicated), can you figure out the matrix A for which $f = f_A$?
- Given a matrix A , what are the domain and codomain of f_A ? What is the value of f_A applied to the standard basis vectors?
- Given two matrices A and B , do you know what the columns of the product AB are in terms of A and the columns of B ? Do you know what the rows of the product AB are in terms of B and the rows of A ?
- What is the dot product $\vec{v} \cdot \vec{w}$ in terms of matrix multiplication?
- Can you determine whether a given set of mathematical objects (matrices, functions, points in \mathbb{R}^n , etc) forms a vector space? Make sure you can show this rigorously, or give an example when something is not a vector space.
- Given a vector \vec{b} , can you find ALL solutions to $A\vec{x} = \vec{b}$? What is this called when $\vec{b} = 0$?
- Can you draw the column space of a given matrix A ?
- Given a set of vectors $\vec{v}_1, \dots, \vec{v}_n$, can you tell when they are linearly independent? Can you tell when they span a given vector space V ? Can you tell when they form a basis? What do these things say about the matrix whose columns are \vec{v}_i ?

- Given a set of vectors $\vec{v}_1, \dots, \vec{v}_n$, how can you find a subset which is a basis for the span?
- Can you state the fundamental theorem of linear algebra?
- If f_A goes from \mathbb{R}^n to \mathbb{R}^m , do you know how many rows and columns A has? Do you know what $C(A)$ is a subspace of? What about $N(A), C(A^T), N(A^T)$? Do you know how the dimensions of these spaces relate?
- Can you find a basis for the null space of A ? For $C(A)$? For $C(A^T)$? For $N(A^T)$?
- Do you know how to tell when two subspaces V, W of \mathbb{R}^n are orthogonal? Do you know how to tell when they are orthogonal complements? If you have two matrices A, B both with m rows, how can you tell when their column spaces are orthogonal?
- Do you know how to find the projection matrix for projection onto a subspace? This includes: do you know how to find a basis for a subspace given by some linear equations?
- Do you know how to calculate the distance of a point from a subspace? How to find the point in a subspace closest to a given point?
- Do you know the geometric meaning of the column space and null space of the projection matrix?
- Do you know the (unsolvable) equation $A\vec{x} = \vec{b}$ corresponding to a line going through a set of k points? Do you know how to find the equation for the line of best fit? Do you know how to find the plane of best fit for points in \mathbb{R}^3 ? What about the paraboloid of best fit?
- Do you know how to find an orthonormal basis for a given subspace?
- Do you know how to calculate determinant using row operations? Do you know how each row operation changes the determinant?
- Do you know how to calculate determinant using the cofactor method?
- Do you know how to find the volume of a parallelepiped?
- Do you know how to find the inverse of a matrix using determinants?
- Do you know how to find the eigenvalues and eigenvectors of a matrix?
- Do you know how to determine when a matrix is diagonalizable? If it is, do you know how to diagonalize it?
- Do you know how to find powers A^k a diagonalizable matrix?

- Do you know the matrix associated to a linear system of differential equations? Do you know how to use it to find a solution? Do you know how the eigenvalues and eigenvectors of the matrix relate to the solution?
- Do you know how to compute the matrix exponential e^M for a square matrix M ?
- Do you know the statement of the spectral theorem? Do you know the dot product of eigenvectors with different eigenvalues **for a symmetric matrix**?
- Do you know the definition of positive semi-definite for matrices? Do you know how to tell when a matrix is positive semi-definite? That is, what can you say about the eigenvalues of a positive semi-definite matrix?
- Do you know how to find the Singular Value Decomposition of a matrix?
- Do you know what properties the matrices U and V satisfy in the SVD? Do you know how these matrices and their columns relate to A , $A^T A$, and AA^T .
- Do you understand the proof of SVD?
- Do you know how to use SVD to find the k -dimensional subspace through the origin best fitting a set of points in \mathbb{R}^n ? (this is PCA). Do you know how to find the projections onto that subspace?
- Do you know how to find the matrix of a linear transformation?
- Do you know how to compute a change of basis matrix?
- Do you know how to tell when a function is linear?
- Do you know what a stochastic matrix is? Do you know what a stochastic vector is? Do you know when a Markov chain converges to a steady state? Do you know the implications of a stochastic matrix being Markov? Can you think of examples of chains that do not converge?
- For a Markov matrix, do you know how the eigenvectors of the matrix relate to steady states = limits of Markov chains?
- Do you know the statements of Perron–Frobenius?
- Do you know what an incidence matrix is? Do you know how its subspaces relate to Kirchoff’s laws? Do you know how to find the number of loops and connected components using only this matrix?
- Do you know what an adjacency matrix is? Do you know how to use it to count the number of nodes directly connected to a given node? Do you know how to count the number of paths of length k from one node to another using the adjacency matrix?

- Do you know how to calculate the page rank of a graph using eigenvectors? Do you know what problems arise if there is no damping? Do you know why damping fixes these problems? Do you know how to calculate a transition matrix using damping?
- Do you know how to setup inequalities coming from constraints to make an LP in standard form? Do you know how to change maximization problems to minimization problems? Do you know how to use the simplex method to find a solution to an LP in \mathbb{R}^n ? (not just \mathbb{R}^2 where you can graph it).