MIDTERM EXAM 1

MATH 312, SECTION 001

Name:

The use of calculators, computers and similar devices is neither necessary nor permitted during this exam. Correct answers without proper justification will *not* receive full credit. Clearly highlight your answers and the steps taken to arrive at them: illegible work will not be graded. You may use both sides of one 8×11 cheat-sheet.

Problem Number	Possible Points	Points Earned
1	25	
2	40	
3	14	
4	21	
Total	100	

Problem 1

[**25** points] The LU decomposition of a matrix A is given as follows:

$$\mathsf{L} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix} \text{ and } \mathsf{U} = \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}.$$

Part a. [3 points] What is A?

Part b. [3 points] Write down the sequence of row operations which takes A to U when performing Gaussian elimination.

Part c. [10 points] Describe how you would solve $\mathsf{A}\mathsf{x} = \left[\begin{smallmatrix}1\\2\\3\end{smallmatrix}\right]$ as two triangular systems[1]. Can we solve these two triangular systems in any order, or must one of them be solved before the other? Explain your answer.

Part d. [5 points] Compute the inverse of U using Gauss-Jordan elimination.

[1] You don't actually have to solve anything: just explain how you'd set up the two triangular systems

Part e. [4 points] Compute A^{-1}, or explain why A is not invertible.

Problem 2

[**40** Points] The matrix B equals MR, where

$$\mathsf{M} = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ 2 & 0 & 2 \end{bmatrix} \text{ and } \mathsf{R} = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

You may also use the fact that

$$\mathsf{M}^{-1} = \begin{bmatrix} 1/4 & -1/4 & 1/4 \\ 1/4 & 1/4 & -1/4 \\ -1/4 & 1/4 & 1/4 \end{bmatrix}$$

Part a. [5 points] Find a basis for the row space $\mathsf{C}(\mathsf{B}^\mathsf{T})$.

Part b. [10 points] Find a basis for the column space $\mathsf{C}(\mathsf{B})$.

Part c. [10 points] Find a basis for the null space $\mathsf{N}(\mathsf{B})$.

Part d. [10 points] Find a basis for the left null space $\mathsf{N}(\mathsf{B}^\mathsf{T})$. **Hint**: you might need M^{-1} for this part.

Part e. [5 points] State the fundamental theorem of linear algebra, and show that B satisfies it.

Problem 3

[**14** Points] Let $\mathsf{B} = \mathsf{M}\mathsf{R}$ be the matrix from Problem 2.

Part a. [7 points] Find *all* solutions to $\mathsf{B}x = \nu$ when $\nu = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$. **Hint:** you *don't* have to compute the RREF of $\begin{bmatrix} \mathsf{B} \mid \nu \end{bmatrix}$ from scratch: you can just use $\begin{bmatrix} \mathsf{R} \mid \mathsf{M}^{-1}\nu \end{bmatrix}$.

Part b. [7 points] Find *all* solutions to $\mathsf{B}x = \nu$ when $\nu = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$. **Hint:** see the hint given for Part a.

Problem 4

[**21** points, 3 points each] In each of the following cases, clearly mark the statement as **true** or **false**. Please also *explain* your answers in order to receive credit for this problem!

a. If a 3×4 matrix has a RREF with only three pivots, then its rows are linearly dependent.

b. If a 3×4 matrix has a RREF with three pivots, then its columns must span $\mathbb{R}^3$.

c. If a matrix A satisfies $Ax = 0$ for some $x \neq 0$ then A cannot be invertible.

d. The set A consisting of the X axis, the Y axis, the line $y = x$ and the line $y = -x$ forms a subspace of $\mathbb{R}^2$.

e. If a vector k lies in the nullspace of A^T and if $Ax = b$ then $A(x + k)$ also equals b.

f. The product of three invertible 3×3 matrices is always invertible.

g. If E is the elementary matrix which adds 3 times Row 1 to Row 2, then E^2 adds 9 times Row 1 to Row 2.

For Scratchwork