MIDTERM EXAM 1

MATH 312, GROUP 001

Version A

Name:

No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity. I certify that all of the work on this test is my own.

Signature:

The use of calculators, computers and similar devices is neither necessary nor permitted during this exam. Correct answers without proper justification will not receive full credit. Clearly highlight your answers and the steps taken to arrive at them: illegible work will not be graded. You may use both sides of one 8×11 cheat-sheet.

Problem Number	Possible Points	Points Earned
1	25	
2	20	
3	15	
4	20	
5	20	
Total	100	

Problem 1

[25 points] Consider the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 0 & -1 & 3 \\ 1 & 2 & -1 & 1 \end{bmatrix}$.

Part a. [5 points] Find the $A = LU$ decomposition (L lower triangular, U upper triangular).

The decomposition of A into the row reduced echelon form $EA = R$ is given by

$$R = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}.$$

Part b. [5 points] Find a basis for the column space of A, $C(A)$.

Part c. [5 points] Find a basis for the nullspace of A, $N(A)$.

Part d. [5 points] Find the complete solution to $A\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$. (Hint: You do not need to reduce the augmented matrix. Use R and E: $A\mathbf{x} = \mathbf{b}$ if and only if $R\mathbf{x} = E\mathbf{b}$).

Part e. [5 points] Find the complete solution to $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (Hint: You do not need to reduce the augmented matrix. Use R and E: $A\mathbf{x} = \mathbf{b}$ if and only if $R\mathbf{x} = E\mathbf{b}$).

Problem 2

[20 points] Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{u}_4 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \mathbf{u}_5 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$.

Part a. [9 points] What is the dimension of the vector space V spanned by $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5$? In other words, how many independent vectors can you find?

Part b. [11 points] Describe how you would write $\mathbf{u}_5$ as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ (you do not need to give the values of the coefficients, simply write how to obtain them).

Problem 3

[15 points] Let V be the line in $\mathbb{R}^2$ defined by $x - y = 0$.

Part a. [5 points] Find a basis for V (that is, a vector in $\mathbb{R}^2$ that spans the line).

Part b. [6 points] Find the projection matrix, P, onto V.

Part c. [4 points] What is the nullspace of P in geometric terms?

Problem 4

[20 points] For a matrix A we know a basis of its nullspace and of its column space:

$$\text{Basis for } N(A) = \{ \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \}, \qquad \text{Basis for } C(A) = \{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \},$$

Part a. [4 points] The linear application given by A goes from $\mathbb{R}^n$ to $\mathbb{R}^m$. Which are the values of n and m?

Part b. [4 points] What is the dimension of $N(A)$ and $C(A)$?

Part c. [4 points] What is the dimension of $C(A^T)$?

Part d. [4 points] Find $C(A^T)$ (you can give equations describing the subspace or a basis) (Hint: You do not need the matrix A for this, use orthogonality).

Part e. [4 points] Find such a matrix A.

Problem 5

[20 points: 2.5 points each] In each of the following cases, clearly mark the statement as **true** or **false**. Please also explain your answers in order to receive credit for this problem.

1. The matrices $A^T A$ and AA^T are always symmetric.

2. In $\mathbb{R}^2$, the orthogonal complement of a line is another line.

3. If a $3x5$ matrix has 3 pivots, then $A\mathbf{x} = \mathbf{b}$ always has infinite solutions.

4. Let A be a square matrix such that $A\mathbf{x} = \mathbf{0}$ for some $\mathbf{x} \neq \mathbf{0}$. Then A can never have an inverse.

5. If A is invertible, then $A(A^T A)^{-1} A^T = I$ always holds (I is the identity matrix).

6. If P_{12} is the matrix that changes row 1 with row 2, then $P_{12}^3 = I$ (I is the identity matrix).

7. The set of points satisfying $y = 3x + z$ and $z + x = 0$ is a subspace of $\mathbb{R}^3$.

8. The set of points satisfying $x^2 + y = 0$ is a subspace of $\mathbb{R}^2$.

Extra space for work:

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