HOMEWORK ASSIGNMENT 8

Name: **Due:** Friday November 2 4pm

Problem 1: Strang 6.4

Which of these matrices ASB will be symmetric with eigenvalues 1 and -1?

$$\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} \qquad \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} \qquad \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$

Justify your answer without just multiplying out the matrices.

Problem 2: Strang 6.4 #6, page 345

Find an orthogonal matrix Q that diagonalizes

$$A = \begin{bmatrix}-2 & 6\\6 & 7\end{bmatrix}$$

What is the diagonal matrix D such that $A = QDQ^T$?

Problem 3: Strang 6.4 #13, page 346

Write A and B in the form

$$\lambda_1 \vec{x}_1 \vec{x}_1^T + \lambda_2 \vec{x}_2 \vec{x}_2^T$$

of the spectral theorem QDQ^T.

$$A = \begin{bmatrix}3 & 1\\1 & 3\end{bmatrix} \qquad B = \begin{bmatrix}9 & 12\\12 & 16\end{bmatrix}$$

(keep $||\vec{x}_1|| = ||\vec{x}_1|| = 1$).

Problem 4: Strang 6.4 #23, page 347

True (with a reason) or false (with example).

1. A matrix with real eigenvalues and n real eigenvectors is symmetric.
2. A matrix with real eigenvalues and n orthonormal eigenvectors is symmetric.
3. The inverse of an invertible symmetric matrix is symmetric.
4. The eigenvector matrix Q of a symmetric matrix is symmetric.

Problem 5: Section 6.4 #26, page 347

What number b in

$$A = \begin{bmatrix} 2 & b \\ 1 & 0 \end{bmatrix}$$

makes $A = QDQ^T$ possible? What number will make it impossible to diagonalize A? What number will make A singular?

Problem 6: Section 6.5 #18, page 360

If $M\mathbf{x} = \lambda\mathbf{x}$ then $\mathbf{x}^T M\mathbf{x} =$______. Why is this number positive when $\lambda > 0$?

Problem 7: Strang 6.5 #20, page 360

Give a quick reason why each of these statements is true:

1. Every positive definite matrix is invertible.
2. The only positive definite projection matrix is $P = I$.
3. A diagonal matrix with positive diagonal entries is positive definite.
4. A symmetric matrix with a positive determinant might not be positive definite!

Problem 8:

Draw a rank 4 flag (doesn't have to be a real country).

Problem 9: Strang 7.1 #3, page 370

These flags have rank 2. Write A and B in any way as $\mathbf{u}_1\mathbf{v}_1^T + \mathbf{u}_2\mathbf{v}_2^T$.

$$A_{Sweden} = A_{Finland} = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 1 & 2 & 1 & 1 \end{bmatrix} \qquad B_{Benin} \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 3 \end{bmatrix}.$$

Problem 10: Strang 7.2 #4, page 379

Compute $A^T A$ and AA^T and their eigenvalues and unit eigenvectors for V and U.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

Check $AV = U\Sigma$ (this decides $\pm$ signs in U). Σ has the same shape as A: 2 by 3.

Problem 11: Strang 7.2 #16, page 380

Suppose A has orthogonal columns $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_n$ of lengths $\sigma_1, \sigma_2, \ldots, \sigma_n$. What are U, Σ and V in the SVD?

Problem 12:

Review up to (and including) section 7.2 and sections 8.1 and 8.2 for the midterm. What would you most like to review?