HOMEWORK ASSIGNMENT 7

Name: **Due:** Wednesday October 24

Problem 1: Strang 6.1 #6, page 299

Find the eigenvalues of A and B and AB and BA:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad AB = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \quad \text{and} \quad BA = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Problem 2: Strang 6.1 #19, page 301

A 3 by 3 matrix B is known to have eigenvalues 0,1,2. This information is enough to find three of these (give the answers where possible):

1. The rank of B.
2. The determinant of $B^T B$.
3. The eigenvalues of $B^T B$.
4. The eigenvalues of $(B^2 + I)^{-1}$.

Problem 3: Strang 6.1 #32, page 302

Suppose A has eigenvalues 0,3,5 with independent eigenvectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

1. Give a basis for the nullspace and a basis for the column space.
2. Find a particular solution to $A\mathbf{x} = \mathbf{v} + \mathbf{w}$. Find all solutions.
3. $A\mathbf{x} = \mathbf{u}$ has no solution. If it did then ___ would be in the column space.

Problem 4:

Diagonalize the matrix

$$A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 4 & 0 \\ 5 & 0 & 0 \end{bmatrix}.$$

What are its eigenvectors and eigenvalues?

Problem 5: Section 6.2 #11, page 315

True or false: If the eigenvalues of a 3 by 3 matrix A are 2,2,5 then the matrix is certainly

1. invertible
2. diagonalizable
3. not diagonalizable

Problem 6: Section 6.2

1. $A^k = X\Lambda^k X^{-1}$ approaches the zero matrix as $k \to \infty$ if and only if every λ has absolute value less than ___. Which of these matrices has $A^k \to 0$?

$$A_1 = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} .6 & .9 \\ .1 & .6 \end{bmatrix}.$$

2. Find Λ and X to diagonalize A_1. What is the limit of Λ^k as $k \to \infty$? What is the limit of $X\Lambda^k X^{-1}$? In the columns of this limiting matrix you see the ___.

Problem 7: Strang 6.2 #26, page 317

Suppose $A\mathbf{x} = \lambda\mathbf{x}$. If $\lambda = 0$ then $\mathbf{x}$ is in the nullspace. If $\lambda \neq 0$ then $\mathbf{x}$ is in the column space. Those spaces have dimensions $(n-r)+r = n$. So why doesn't every square matrix have n linearly independent eigenvectors?

Problem 8: Strang 6.3 #8, page 333

The rabbit population shows fast growth (from $6r$) but loss to wolves (from $-2w$). The wolf population always grows in this model ($-w^2$ would control wolves):

$$\frac{dr}{dt} = 6r - 2w \qquad \text{and} \qquad \frac{dw}{dt} = 2r + w. \tag{1}$$

Find the eigenvalues and eigenvectors. If $r(0) = w(0) = 30$ what are the populations at time t? After a long time, what is the ratio of rabbits to wolves?

Problem 9: Strang 6.3 #9, page 335

(a) Write (4,0) as a combination of $c_1\vec{x}_1 + c_2\vec{x}_2$ of these two eigenvectors of A:

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = i\begin{bmatrix} 1 \\ i \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -i \end{bmatrix} = -i\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

Problem 10: Strang 6.3 #25, page 335

Put $A = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$ into the infinite series to find e^{At}. First compute A^2 and A^n:

$$e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} t & 3t \\ 0 & 0 \end{bmatrix} + \cdots = \begin{bmatrix} e^t & ? \\ 0 & ? \end{bmatrix}$$

Problem 11:

Read Sections 6.4 to 7.2. Which concept was most difficult to you?