HOMEWORK ASSIGNMENT 6

Name: **Due:** Friday October 17

Problem 1: Strang 5.1 #1, page 254

If a 4 by 4 matrix has det $A = \frac{1}{2}$, find det $(2A)$ and det $(-A)$ and det (A^2) and det (A^{-1}).

Problem 2: Strang 5.1 #3, page 254

True or false, with a reason if true or a counterexample if false:

1. The determinant of $I + A$ is $1 + \det(A)$.
2. The determinant of ABC is $|A||B||C|$.
3. The determinant of $4A$ is $4|A|$.
4. The determinant of $AB - BA$ is zero. Try an example with $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Problem 3: Strang 5.1 #22, page 256

From $ad - bc$, find the determinants of A and A^{-1} and $A - \lambda I$:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}.$$

Which two numbers λ lead to det $(A - \lambda I) = 0$? Write down the matrix $A - \lambda I$ for each of those numbers λ (it should not be invertible).

Problem 4: Strang 5.1 #27, page 257

Compute the determinants of these matrices by row operations

$$A = \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \\ d & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}.$$

Problem 5: Section 5.2 #2, page 267

Compute the determinants of A, B, C, D. Are their columns independent?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}.$$

Problem 6: Section 5.2 #13, page 268

The n by n determinant C_n has 1's above and below the main diagonal:

$$C_1 = |0| \qquad C_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \qquad C_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \qquad C_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}.$$

1. What are these determinants C_1, C_2, C_3, C_4?
2. By cofactors find the relation between C_n and C_{n-1} and C_{n-2}. Find C_{10}.

Problem 7: Strang 5.3 #18, page 285

1. The corners of a triangle are $(2, 1)$ and $(3, 4)$ and $(0, 5)$. What is the area?
2. Add a corner at $(-1, 0)$ to make a lopsided region (four sides). Find the area.

Problem 8: Strang 5.3 #27, page 285

Polar coordinates satisfy $x = r\cos\theta$ and $y = \sin\theta$. Polar area is $J\,dr\,d\theta$:

$$J = \begin{vmatrix} \partial x/\partial r & \partial x/\partial\theta \\ \partial y/\partial r & \partial y/\partial\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}.$$

The two columns are orthogonal. Their lengths are _____. Thus $J =$_____.

Problem 9:

Read Sections 6.1 to 6.4 (Eigenvalues). Which concept was most difficult to you?