HOMEWORK ASSIGNMENT 4

Name: **Due:** Wednesday September 26

Problem 1: Strang 4.1 #3,(c)(d)(e), page 202

Construct a matrix with the required property or say why that is impossible:

1. $A\mathbf{x} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ has a solution and $A^T \begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$
2. Every row is orthogonal to every column (A is not the zero matrix)
3. Columns add up to a column of zeros, rows add to a row of 1's.

Problem 2: Strang 4.1 #4, page 202

If $AB = 0$ then the columns of B are in the _____ of A. The rows of A are in the _____ of B. With $AB = 0$, why can't A and B be 3 by 3 matrices of rank 2?

Problem 3: Strang 4.1 #14, page 203

The floor V and the wall W are not orthogonal subspaces, because they share a nonzero vector (along the line where they meet). No planes V and W in $\mathbb{R}^3$ can be orthogonal! Find a vector in the column spaces of both matrices:

$$A = \begin{bmatrix}1 & 2\\1 & 3\\1 & 2\end{bmatrix}, \qquad B = \begin{bmatrix}5 & 4\\6 & 3\\5 & 1\end{bmatrix}$$

This will be a vector $A\mathbf{x}$ and also $B\hat{\mathbf{x}}$. Think 3 by 4 with the matrix $[A\ \ B]$.

Problem 4: Strang 4.1 #17, page 204

If S is the subspace of $\mathbb{R}^3$ containing only the zero vector, what is $S^\perp$? If S is spanned by $(1,1,1)$, what is $S^\perp$? If S is spanned by $(1,1,1)$ and $(1,1,-1)$, what is a basis for $S^\perp$?

Problem 5: Section 4.2 #1, page 214

Find the projection, $\mathbf{p}$, of $\mathbf{b}$ onto the line through $\mathbf{a}$. Check that $\mathbf{e}$ ($\mathbf{e} = \mathbf{b} - \mathbf{p}$) is perpendicular to $\mathbf{a}$:

$$(a)\ \mathbf{b} = \begin{bmatrix}1\\2\\2\end{bmatrix} \text{ and } \mathbf{a} = \begin{bmatrix}1\\1\\1\end{bmatrix} \qquad (b)\ \mathbf{b} = \begin{bmatrix}1\\3\\1\end{bmatrix} \text{ and } \mathbf{a} = \begin{bmatrix}-1\\-3\\-1\end{bmatrix}$$

Problem 6: Section 4.2 #3, page 215

In the previous problem (Problem 5), find the projection matrix $P = \mathbf{a}\mathbf{a}^T/\mathbf{a}^T\mathbf{a}$ onto the line through each vector $\mathbf{a}$. Verify in both cases that $P^2 = P$. Multiply $P\mathbf{b}$ in each case to compute the projection $\mathbf{p}$.

Problem 7: Strang 4.2 #16, page 216

What linear combination of $(1, 2, -1)$ and $(1, 0, 1)$ is closest to $\mathbf{b} = (2, 1, 1)$?

Problem 8: Strang 4.2 #19, page 216

To find the projection matrix onto the plane $x - y - 2z = 0$, choose two vectors in that plane and make them the columns of A. The plane will be the column space of A! Then compute $P = A(A^TA)^{-1}A^T$.

Problem 9: Strang 4.3 #1, page 229

With $b = 0, 8, 8, 20$ at $t = 0, 1, 3, 4$, set up and solve the normal equations $A^TA\hat{\mathbf{x}} = A^T\mathbf{b}$. For the best straight line in Figure 4.9a, find its four heights p_i and four errors e_i. What is the minimum value $E = e_1^2 + e_2^2 + e_3^2 + e_4^2$?

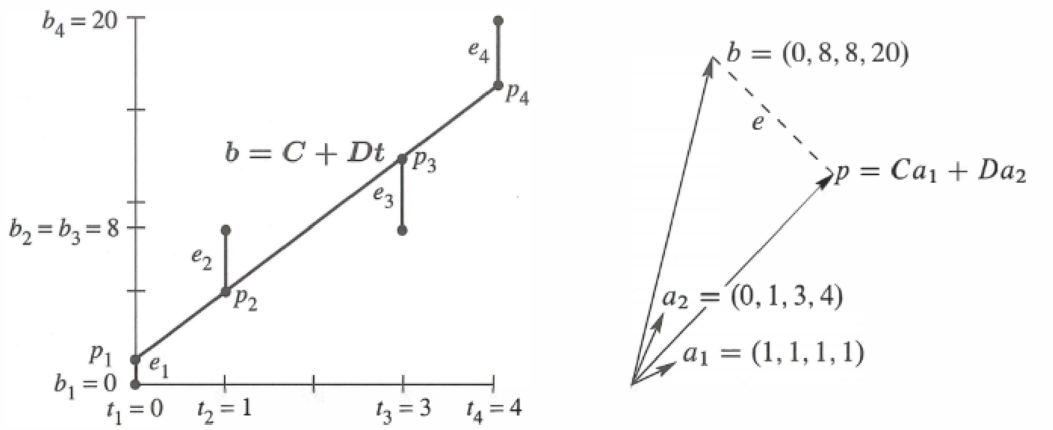

Figure 4.9: **Problems 1–11**: The closest line $C + Dt$ matches $Ca_1 + Da_2$ in $\mathbf{R}^4$.

Problem 10: Strang 4.3 #21, page 231

Which of the four subspaces contains the error vector $\mathbf{e}$? Which contains $\mathbf{p}$? Which contains $\hat{\mathbf{x}}$? What is the nulspace of A?

Problem 10:

Study the material for the first midterm exam (Chapters 1, 2, 3 and Sec. 4.1, 4.2 of Chapter 4). Which concept would you most like to review?