HOMEWORK ASSIGNMENT 3

Name: **Due:** Thursday September 20

Problem 1: Strang 3.3 #10, page 159

Construct a 2 by 3 system $A\mathbf{x} = \mathbf{b}$ with particular solution $\mathbf{x_p} = (2, 4, 0)$ and homogeneous solution $\mathbf{x_n} =$ any multiple of $(1, 1, 1)$.

Problem 2: Strang 3.3 #13, page 160

Explain why these are all false:

1. The complete solution is any linear combination of $\mathbf{x_p}$ and $\mathbf{x_n}$.
2. A system $A\mathbf{x} = \mathbf{b}$ has at most one particular solution.
3. The solution $\mathbf{x_p}$ with all free variables zero is the shortest solution (minimum length $\|\mathbf{x}\|$). Find a 2 by 2 counterexample.
4. If A is invertible there is no solution $\mathbf{x_n}$ in the nullspace.

Problem 3: Strang 3.3 #33, page 162

The complete solution to $A\mathbf{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find A.

Problem 4: Strang 3.4 #1, page 175

Show that $\mathbf{v_1}$, $\mathbf{v_3}$, $\mathbf{v_3}$ are independent but $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_4}$ are dependent:

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{v_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{v_3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{v_4} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}.$$

Solve $c_1\mathbf{v_1} + c_2\mathbf{v_2} + c_3\mathbf{v_3} + c_4\mathbf{v_4} = \mathbf{0}$ or $A\mathbf{x} = \mathbf{0}$. The $\mathbf{v}$'s go in the columns of A.

Problem 5: Section 3.4 #2, page 175

(Recommended) Find the largest possible number of independent vectors among

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{v_2} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad \mathbf{v_3} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{v_4} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad \mathbf{v_5} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{v_6} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Problem 6: Section 3.4 #9, page 176

Suppose $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_4}$ are vectors in $\mathbb{R}^3$.

1. These four vectors are dependent because _____.
2. The two vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ will be dependent if _____.
3. The vectors $\mathbf{v_1}$ and $(0,0,0)$ are dependent because _____.

Problem 7: Strang 3.4 #16, page 176

Find a basis for each of these subspaces of $\mathbb{R}^4$:

1. All vectors whose components are equal.
2. All vectors whose components add to zero.
3. All vectors that are perpendicular to $(1,1,0,0)$ and $(1,0,1,1)$.
4. The column space and the nullspace of I (4 by 4).

Problem 8: Strang 3.5 #3, page 190

Find a basis for each of the four subspaces associated with A:

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Problem 9:

All you know about a 3×5 matrix A is that it has rank 2. Compute $\dim \mathrm{N}(A) - \dim \mathrm{N}(A^T) + \dim \mathrm{C}(A) - \dim \mathrm{C}(A^T)$. Explain how you got your answer.

Problem 10:

Read Chapter 4 (Strang). Which concept was more confusing for you?