HOMEWORK ASSIGNMENT 2

Name: **Due:** Wednesday September 12

Problem 1: Strang 2.1 #16, page 43

(a) What 2 by 2 matrix R rotates every vector by 90°? R times $\begin{bmatrix} x \\ y \end{bmatrix}$ is $\begin{bmatrix} y \\ -x \end{bmatrix}$.

(b) What 2 by 2 matrix R^2 rotates every vector by 180°?

Problem 2: Strang 2.3 #8, page 66

The ***determinant*** of $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is det $M = ad - bc$. Subtract l times row 1 from row 2 to produce a new M^*. Show that det M* = det M for every l. When $l = c/a$, *the product of the pivots equals the determinant*: $a(d - lb)$ equals $ad - bc$.

Problem 3: Strang 2.3 #17, page 67

The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4)$ and $(2, 8)$ and $(3, 14)$. Find and solve a matrix equation for the unknowns (a, b, c).

Problem 4: Strang 2.6 #7, page 105

What three elimination matrices E_{21}, E_{31}, E_{32} put A into its upper triangular form $E_{32}E_{31}E_{21}A = U$? Multiply by E_{32}^{-1}, E_{31}^{-1} and E_{21}^{-1} to factor A into L times U:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}, \qquad L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}.$$

Problem 5: Section 2.7

Consider a matrix that satisfies $A^T = A^{-1}$, i.e., $AA^T = A^TA = I$. If we denote the columns of A by $a_1, a_2, \ldots, a_n$, show that

$$\vec{a_i} \cdot \vec{a_i} = 1 \quad \text{for } i = 1, 2, \ldots, n,$$
$$\vec{a_i} \cdot \vec{a_j} = 0 \quad \text{for } i \neq j,$$

that is, different columns are perpendicular and all columns have norm 1. The same happens to the rows. These matrices are called *orthogonal*.

Problem 6: Section 3.1

Which of the following subsets of $\mathbb{F}$ (the vector space of functions from $\mathbb{R}$ to $\mathbb{R}$) are actually subspaces? Explain your answer.

1. The set of functions f such that $f(0) = 1$.
2. The set of functions f such that $f(0) = 0$.
3. The set of polynomials in a single variable.
4. The set of functions f such that $f(-x) = -f(x)$ for all real numbers x (these are called **odd** functions.
5. The set of functions f such that $f(-x) = f(x)$ for all real numbers x (these are called **even** functions.
6. The set of functions f that are either even, odd, or both (this is called the **union** of the sets of even and odd functions).

Problem 7: Strang 3.1 #20, page 133

For which right sides (find a condition on b_1, b_2, b_3) are these systems solvable?

$$(a) \begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \qquad (b) \begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Problem 8: Strang 3.1 #23, page 134

(Recommended) If we add an extra column $\mathbf{b}$ to a matrix A, then the column space gets larger unless _____ . Give an example where the column space gets larger and an example where it doesn't. Why is $A\mathbf{x} = \mathbf{b}$ solvable exactly when the column space *doesn't* get larger - it is the same for A and $[A\ \mathbf{b}]$?

~~Problem 9: Strang 3.2 #18, page 144~~

~~Construct a matrix whose column space contains $(1, 1, 0)$ and $(0, 1, 1)$ and whose nullspace contains $(1, 0, 1)$ and $(0, 0, 1)$.~~

Problem 9: Strang 3.2 #17, page 144

Construct a matrix whose column space contains $(1, 1, 5)$ and $(0, 3, 1)$ and whose nullspace contains $(1, 1, 2)$.

Problem 10:

Read Chapter 3 (Strang). Which concept was more confusing for you?